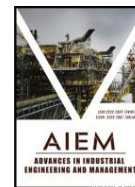




# Advances In Industrial Engineering And Management (AIEM)

DOI: <http://doi.org/10.7508/aiem.01.2020.51.56>



## ARTICLE

# DEVELOPMENT OF A SOLUTION TECHNIQUE FOR THE FACILITY LOCATION AND STEP-FIXED CHARGE SOLID TRANSPORTATION PROBLEM

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## ARTICLE DETAILS

## ABSTRACT

### Article History:

Received 12 June 2020  
Accepted 23 July 2020  
Available online 28 August 2020

In this paper, a new variant of the Solid Transportation Problem (STP) that incorporates both facility location and Step-Fixed Charge Solid Transportation Problem (SFCSTP) is presented with applications in logistics. It integrates decisions of diverse planning horizons: operational, tactical and strategic. The problem is termed Step-Fixed Charge Solid Location and Transportation Problem (SFCSLTP). Its objective function behaves like a step-function with breakpoints. The SFCSLTP considers three possible scenarios of breakpoints. The first is termed scenario 1, occurring at the minimum demand point. The second scenario at the maximum demand (scenario 2). The third scenario (scenario3) occurs at a point in between the maximum and minimum demand requirements. Benchmark data obtained from literature was extended for experimentation. The solution to the SFCSLTP was obtained using CPLEX solver. Results from the experimentation with scenarios 1, 2, and 3 respectively confirmed the analytical prediction of a possible decrease in the minimum cost when the breakpoint tends towards the maximum demand point. In addition, experimentation results offer new insights into transportation models where the objective function behaves as step-function, and also about computation efficiency of such models with solvers such as CPLEX. Managers involved with logistic planning will find this model useful.

### KEYWORDS

Piecewise-Linear Cost, Location-Fixed Cost, Route-Fixed Cost, Break-Point Scenarios, CPLEX.

## 1. INTRODUCTION

The Solid Transportation Problem (STP) described as multi-index simple transportation problem was introduced by Schell[1]. STP is essentially a transportation problem from source to destination which takes into account conveyance or transport capacities during distribution decisions. A method to solve the STP was developed by Haley[2]. An application of the solid transportation problem has been creating a shipment plan that guides operational managers on the number of products to move from production or warehouse locations while selecting from different transportation sources to decrease the total of fixed and variable transportation costs. Distribution problems such as the STP, and most especially those that involve joint consideration of various individual optimization decisions has lately been extended by several authors in the literature. For instance, the Fixed Charge Solid Transportation Problem (FCSTP) and the Step-Fixed Charge Solid Transportation Problem (SFCSTP) described by Saneii[3], and also the Facility location and Solid Transportation problem modelled by Das et al.[3-4]. These optimization decisions could be daily operational types, medium tactical decisions, and/or the more strategic decisions such as facility location. The main aim of planning these decisions together is to prevent individual sub-optimal decisions.

The Solid Transportation problem is a basic extension of the simple Transportation Problem (TP) to capture real-world shipment scenarios.

Some transportation problem variants being explored by researchers are the Fixed Charge Transportation Problem (FCTP), Fixed Charge Solid Transportation problem (FCSTP) studied by Das et al.[4-6]. In addition, some other variants of the TP, are the Step-Fixed Charge Solid transportation problem (SFCSTP) studied by Saneii[3], the raw material blending and transportation problem modelled by Kundu, the green transportation problems considered by Qu et al. and fuzzy distribution problems studied by Liu et al.[7-9].

Both the FCSTP and SFCSTP are distribution problems which involve m-sources, n-destinations and k-conveyances indicated that both FCSTP and SFCSTP seek to determine the quantities that the chosen capacitated k-conveyances will be able to ship from the m-sources to n-destinations under a mix of route fixed charges at minimum cost. In problems involving more than one fixed charges such as the SFCSTP and SFCSTP, the fixed charges are represented by the vehicle cost of conveying different volumes of the load[3]. As noted by Oyewole[10], these are incurred either through duties, taxes or vehicle costs of different volumes of load transported. The number of fixed charges depends on the number of breakpoints in the step-function or basically, the toll policies or shipment constraints encountered.

Facility Location Problem (FLP) mostly involves making long term decisions. This involves selecting, from a set of possible locations, the optimal set of locations from which to service a set of customers

considered capacitated plant location models of the multi-objective, multiproduct type and large scale single-source capacitated models studied discrete facility location models with exact and heuristics solution methods[11-17]. Facility location considering stochastic inputs were considered by Oyewoleet al. considered a row perturbation heuristic to solve the capacitated fixed charge and facility location problem[10]. A solid transportation problem which considers facility location decisions in its transportation decisions was modelled and solved by Daset al.[4]. They developed a locate-allocate heuristic to solve the problem. However, they did not consider the reality of fixed charges in their models.

In this paper, we present a distribution problem that simultaneously optimizes facility location and step-fixed charge solid transportation problem. We have termed this problem as Step-Fixed Charge Solid Location and Transportation Problem (SFCSLTP). The objective of the SFCSLTP is to minimize total transportation and location costs by determining the optimal allocations from open locations through open routes by a set of conveyances. In addition, we consider a special case of the SFCSLTP as shown in Figure 1, without loss of generalization. This considers decisions of incurring either one or both of the fixed charges along the transportation route. In order to solve this problem, we utilize the CPLEX mixed-integer dynamic solver which utilizes the branch and cut algorithm to search for optimality.

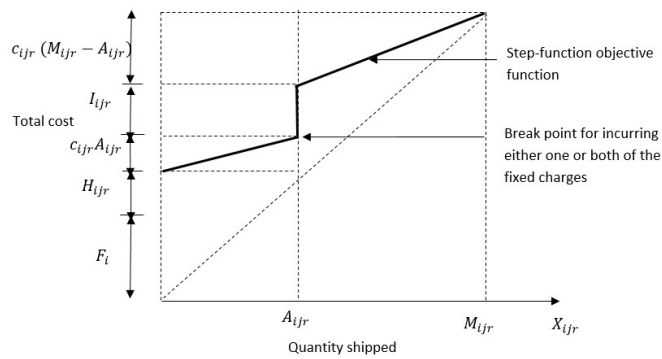


Figure 1: Representation of the objective cost structure for SFCSLTP.

2. MATERIALS AND METHODS

We formulate the SFCSLTP as a Mixed Integer Programming (MIP) problem, with m sources, n destinations, and a conveyances. This extends the model of SFCSTP as presented by Sanei et al. to include fixed costs of facility location[3]. In their SFCSTP a single product is to be shipped from a set of locations, with a possibility of incurring more than one route fixed charges, to a number of demand points using a set of transport mediums. The capacity of each location to supply products in SFCSTP is simply determined by the route fixed charges and variable costs, and also the problem capacities. However, in our SFCSLTP, fixed location costs, route fixed charges and variable route costs and problem capacities are jointly used in determining whether the locations will be opened or closed for shipment along the transportation route.

2.1 Model assumptions for the SFCSLTP

The following basic assumptions are made in our model formulation:

1. Deterministic costs.
2. Two echelon distribution problem.
3. Fixed location costs, route variable and two fixed charge cost without loss of generalization.
4. One planning period and single item distribution problem.

2.2 Model development

The parameters and variables used in the model formulation are given

below.

(1) Parameters

i: Index for sources or locations (plants, warehouses, depots etc.).

j: Index for destinations (customers, other warehouses etc.).

r: Index for conveyances (or Transportation mediums).

m: Number of sources.

n: Number of destinations.

a: Number of conveyances.

$c_{ijr}$  : Variable cost of shipment on route (i,j) using conveyance r.

$S_i$  : Capacity of source  $i \forall i=1 \dots m$ .

$D_j$ : Demand at Destination  $j \forall j=1 \dots n$ .

$T_r$  : Capacity for the conveyance  $\forall r=1 \dots a$ .

$F_i$ : Fixed cost of opening the facility at location i.

$A_{ijr}$ : Breakpoint determining if incurring either one or both fixed charges on route (i,j,r)

$H_{ijr}$  : First level of the fixed cost incurred for shipping through route(i,j) using conveyancer before breakpoint  $A_{ijr}$

$I_{ijr}$  : Second fixed cost incurred for shipping through route (i,j) using conveyance r after breakpoint  $A_{ijr}$

(2) Decision variables

$x_{ijr}$ : Quantity of products transported from source(i) to destination(j) using conveyance (r).

$y_i$  : Location variable for setting source (i) as either opened or closed.

$z_{ijr}$ : First fixed charge variable in selecting to incur the first fixed charge on route(i,j,r).

$w_{ijr}$ : Second fixed charge variable in selecting to incur the second fixed charge on route(i,j,r).

A mathematical model of the FCSTP is described below.

(3) Objective function (minimum cost function) for SFCSLTP

Min (Z):

$$\sum_{i=1}^m F_i y_i + \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^a c_{ijr} x_{ijr} + \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^a H_{ijr} z_{ijr} + \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^a I_{ijr} w_{ijr} \quad (1)$$

Subject to

$$\sum_{j=1}^n \sum_{r=1}^a x_{ijr} \leq S_i y_i \quad \forall i = 1 \dots m \quad (2)$$

$$\sum_{i=1}^m \sum_{r=1}^a x_{ijr} = D_j \quad \forall j = 1 \dots n \quad (3)$$

$$\sum_{i=1}^m \sum_{j=1}^n x_{ijr} \leq T_r \quad \forall r = 1 \dots a \quad (4)$$

$$x_{ijr} \leq M_{ijr} z_{ijr} \quad \forall i = 1 \dots m, \quad \forall j = 1 \dots n, \quad \forall r = 1 \dots a \quad (5)$$

$$x_{ijr} - A_{ijr} \leq M_{ijr} w_{ijr} \quad \forall i = 1 \dots m, \quad \forall j = 1 \dots n, \quad \forall r = 1 \dots a \quad (6)$$

$$M_{ijr} = \min (S_i, D_j, T_r )$$

$$x_{ijr} \geq 0 \quad \forall i = 1 \dots m, \quad \forall j = 1 \dots n, \quad \forall r = 1 \dots a \quad (7a)$$

$$z_{ijr} = \begin{cases} 1 & x_{ijr} > 0 \\ 0 & \text{Otherwise} \end{cases} \quad \forall i = 1 \dots m, \quad \forall j = 1 \dots n, \quad \forall r = 1 \dots a \quad (7b)$$

$$w_{ijr} = \begin{cases} 1 & x_{ijr} > A_{ijr} \\ 0 & \text{Otherwise} \end{cases} \quad \forall i = 1 \dots m, \quad \forall j = 1 \dots n, \quad \forall r = 1 \dots a \quad (7c)$$

$$y_i = 0 \text{ or } 1 \quad \forall i = 1 \dots m \quad (7d)$$

Expression (1) is the minimum cost or objective function. The first term is the facility location cost, the second term is the route variable cost per conveyance type and the third term is the two routes fixed-charge cost per conveyance type.

Constraint (2) ensures that the supply capacity is not exceeded.

Constraint (3) is the demand to be met at each destination.

Constraint (4) ensures that conveyance capacity is not exceeded.

Constraint (5) refers to an upper bound limit on the first route fixed charge.

Constraint (6) refers to an upper bound limit on the second route fixed charge.

Constraint(7a) refers to the non-negativity constraint for the continuous variables.

Constraints (7b and 7c) refer to the binary constraints for the fixed charge and step-fixed charge requirements respectively.

Constraint (7d) refers to the facility location requirement.

### 2.3 Solving the FCSLTP using CPLEX

Wolsey reported on the formulation and application of the branch and cut algorithm to solve a variety of Mixed-integer problems (MIP)[18]. According to Studio et al., the branch and cut are available in commercial development platform for modelling and solving combinatorial problems such as CPLEX. Our SFCSLTP is formulated as a MIP and solved using the MIP dynamic optimizer tool of the CPLEX[19]. The dynamic optimizer tool has the capacity to serially launch a variety of exact solutions to solve a MIP. In order to solve a minimization MIP, the dynamic optimizer uses the continuous relaxation of integrality constraints to obtain a lower bound from which different cuts are applied to improve on the lower bounds obtained. Some cuts used are the mixed-integer rounding cuts, cover cuts and Gomory fractional cut[19].

### 2.4 Some breakpoints analysis

In this section, we provide some insights on choosing different breakpoints, how this might possibly affect the minimum cost solution desired and also some interpretations that could be derived. The breakpoint for deciding whether to incur the first fixed charge or the second fixed charge could have several interpretations.

The manufacturer or distributor, who has been termed as the operating or planning personnel in this paper, naturally wants to ship more products to the customers to make more money. However, the operating

personnel are also constrained by variable and fixed costs such as toll fees or inhibiting fees that prevent shipping more to the customers. While trying to minimize total operating costs, the operating personnel are likely to fall into the following three possible scenarios as given below.

#### (1) Scenario 1

The first scenario occurs when the breakpoint is at the minimum of all their demand requirements. At this position, the planners or operating personnel have very minimum control in deciding the fixed charges incurred. There is a high likely-hood that the solution possibly incurs more of both fixed charges and expected to become the most expensive solution.

Mathematically this is represented with equation 8 below:

Given the Minimum of all the demand as  $\min(D_j) \quad \forall j = 1 \dots n$

$$A_{ijr} = \min(D_j) \quad \forall i = 1 \dots m, \quad \forall j = 1 \dots n, \quad \forall r = 1 \dots a \quad (8)$$

#### (2) Scenario 2

The second scenario occurs when the breakpoint is just at the maximum of the demand requirements. This could imply that the operating personnel have a lot of control over deciding which fees or charges to incur. This scenario tends towards the Fixed Charge Solid Location and Transportation problem (FCSLTP). The solution to this scenario tends towards incurring one fixed charge and is expected to be the cheapest solution for SFCSLTP.

Mathematically this is represented with equation 9 below:

Given the Maximum of all the demand as  $\max(D_j) \quad \forall j = 1 \dots n$

$$A_{ijr} = \max(D_j) \quad \forall i = 1 \dots m, \quad \forall j = 1 \dots n, \quad \forall r = 1 \dots a \quad (9)$$

#### (3) Scenario 3

This scenario occurs when the breakpoint is between the minimum and maximum of all the demand. This means the operating or planning personnel can either have the most expensive or cheapest solution. The solution obtained would largely be determined by the position of the breakpoint. In situations where the operating or planning personnel cannot influence or negotiate the breakpoint within this scenario (third scenario), they are constrained to operate at any breakpoint specified to them. However, in situations where the planners or operating personnel can influence the breakpoint such as negotiating with the government on policies for fixed charges such as toll fees, they possibly can reduce their costs by looking out for a breakpoint that would tend towards the reduced cost for them.

Mathematically this is represented with equation 10:

$$\min(D_j) \text{ and } \max(D_j)$$

$$A_{ijr} : \min(D_j) < A_{ijr} < \max(D_j) \quad \forall i = 1 \dots m, \quad \forall j = 1 \dots n, \quad \forall r = 1 \dots a \quad (10)$$

### 2.5 Computational study

To demonstrate the utility of the model, assess the effectiveness (objective value of OP) of using CPLEX and the effect of different breakpoint positions, we conducted a number of experiments using reference problems obtained in the literature (under section 2.5.1 below). The Original problem (OP) was coded on the ECLIPSE development platform using Java and IBM ILOG CPLEX 12.8 as the MIP solver. We have used a windows 8.1 operating system with 6 GB Random Access Memory(RAM) for the computation experiments.

(1) Data generation for the problem sizes

We have utilized the random Benchmark data used by Sanei et al.[3]. This basically considers uniformly distributed data randomly generated as integers in a unit square coordinate U[a,b]. However, for the breakpoint (A<sub>ijr</sub>) and second level fixed charge (I<sub>ijr</sub>), we chose different random values from that of Sanei et al.[3]. A method for generating facility location cost presented by Gadegaard et al. was used[20,21]. This is given as  $F_i = U(0,90) + \sqrt{S_i} U(100,110)$ .

A total of 5 problem sizes were generated. Each problem was solved under 5 different breakpoints. A total of 25 problems were solved and the results presented in Table 1. The data distributions used in generating the parameters are given in Table 2 below.

(2) Data generation for the different breakpoints

We used 5 different values for the breakpoint (A<sub>ijr</sub>).

This was generated using the formula :  $A_{ijr} = \min(D_j) + x \% \text{ Diff}$   
Where Diff=  $\max(D_j) - \min(D_j)$  and  $x = 0\%$  to  $100\%$ .

The breakpoints are given below:

- $A_{ijr} = \min(D_j) + 0 \% \text{ Diff} = \min(D_j)$  (this is termed break 0%)
- $A_{ijr} = \min(D_j) + 20 \% \text{ Diff}$  (this is termed break 20%)
- $A_{ijr} = \min(D_j) + 50 \% \text{ Diff}$  (this is termed break 50%)
- $A_{ijr} = \min(D_j) + 80 \% \text{ Diff}$  (this is termed break 80%)
- $A_{ijr} = \min(D_j) + 100\% \text{ Diff} = \text{Max}(D_j)$  (this is termed break 100%)

**Table1:** Problem sizes and number of instances used for experimentation.

Problem size No.	Problem size m×n×a
1	5×5×2
2	5×8×2
3	7×7×3
4	7×10×3
5	8×8×4
6	8×12×4
7	10×10×5
8	12×12×6
9	10×15×7
10	15×15×7

**Table 2:** Parameter distribution used for experimentation.

Parameter distribution
$S_i$ U(200, 400)
$D_j$ U(50, 100)
$T_r$ U(800, 1800)
$c_{ijr}$ U(20, 150)
$H_{ijr}$ U(200, 600)
$I_{ijr}$ U(400, 800)
$F_i = U(0, 90) + \sqrt{S_i} U(100, 110)$
$M_{ijr} = \min(S_i, D_j, T_r)$
$A_{ijr} = \min(D_j) + x \% \text{ Diff}$
Where $x = 0\%$ to $100\%$ and Diff = $\max(D_j) - \min(D_j)$

**2.6 Experimentation**

The following tests were conducted in our computational study: We solved for the minimum cost of the SFCSLTP across different values of A<sub>ijr</sub> as described in section 2.5. This was done to observe the magnitude of the cost as it varies across the breakpoints. We possibly expect a decreasing trend across the five (5) breakpoints as indicated in sections 2.4 and 2.5.2. However, the rate of decrease with different breakpoint may not easily be observable analytically, except experimentally conducted. Therefore, we also conducted the percentage decrease

selecting two breakpoints closer to each other.

For instance, the percentage decrease between Break 0% and Break 20 % is computed as  $((\text{Break } 20\% - \text{Break } 0\%) / (\text{Break } 0\%))\%$  and termed as Break((20%)/(0%))

Similarly, other definitions follow as below:

$((\text{Break } 50\% - \text{Break } 20\%) / (\text{Break } 20\%))\%$  is termed Break( (50%)/(20%))

$((\text{Break } 80\% - \text{Break } 50\%) / (\text{Break } 50\%))\%$  is termed Break( (80%)/(50%))

$((\text{Break } 100\% - \text{Break } 80\%) / (\text{Break } 80\%))\%$  is termed Break((100%)/(80%))

We take the magnitude of this computation. A lower value will indicate a low percentage cost decrease while a higher value will indicate a higher cost decrease. The Higher cost decrease region is therefore desirable for a firm wishing to operate at a reduced cost.

We recorded the computation time and observed how it varied with an increase in problem size and increase in breakpoint value A<sub>ijr</sub>. This was performed in order to note any computation time complexity with the solution method and the breakpoints.

**3. RESULTS AND DISCUSSION**

Table 3 below shows the minimum cost (objective function) obtained across different values of A<sub>ijr</sub>.

**Table 3:** Minimum cost obtained across different scenarios of A<sub>ijr</sub>.

Problem size	Break 0%	Break 20%	Break 50%	Break 80%	Break 100%
5×5×2	26901.472	26529.472	26497.888	24385.947	23793.610
5×8×2	34874.960	34074.966	32362.505	30561.045	30204.633
7×7×3	25334.289	24740.776	23983.249	22757.615	21997.375
7×10×3	36081.464	35418.514	34018.128	32324.534	30699.434
8×8×4	24645.013	24146.859	22865.131	21302.969	20954.468
8×12×4	40154.570	38970.176	37114.988	35710.548	34939.098
10×10×5	32193.046	31432.784	30837.697	29018.006	28503.250
12×12×6	36963.756	36440.510	35256.815	34188.738	33064.884
10×15×7	44044.271	42961.852	40960.735	38636.677	37622.358
15×15×7	41581.830	39581.665	38379.001	36797.763	36468.560

Under Experimentation (A), we observe a decreasing trend of minimum cost across the different breakpoints (i.e. from A<sub>ijr</sub> =min(D<sub>j</sub>) to A<sub>ijr</sub> =max(D<sub>j</sub>)). The decreasing trend follows logically from our analysis in section 2.4. This would imply that a minimum operating cost would occur at a break-point at the maximum demand. Therefore, a firm confronted with facility location and two fixed charges (without loss of generality) cost minimization problem wishing to operate at the cheapest cost must be able to influence policymakers towards a break-point at their maximum demand. This was indicated under scenario1. Moreover, fixing the breakpoint at the minimum demand point gives the most expensive minimum cost as indicated in Table 3. This will naturally not be a desirable option for a firm wishing to minimize their operating cost and maximizing their product outputs to meet demand.

If it will not be feasible for a firm to fix their breakpoint at the maximum demand point (cheapest solution) and the minimum demand point (most expensive solution) then it will be necessary for the firm to know where their breakpoint can be in the downward cost trend to achieve a reduced operating cost. This will be necessary for negotiations with policymakers such as those involved with the fixing of toll fees and duty payments. The policymakers will naturally prefer a break-point (possibly towards the minimum demand of firms) that maximizes their revenue. In order for a firm to observe wherein the downward cost trend it may be necessary to negotiate a breakpoint, we compute the percentage decrease in cost using the break-points described in section 2.5.2.

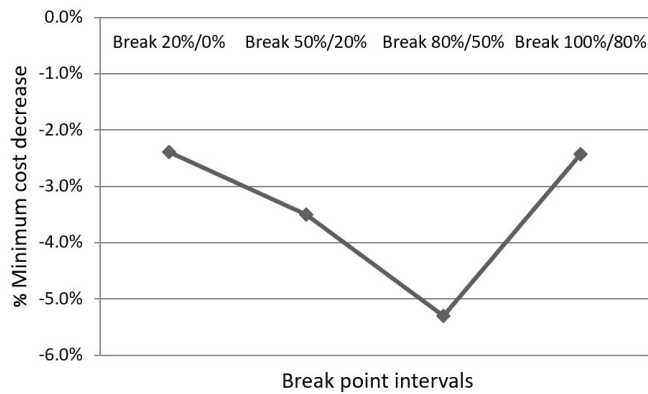
We present the percentage minimum cost decrease in Table 4 below.



**Table 4:** Percentage decrease (%) based on different breakpoint intervals.

Problem size	Break ((20%)/(0%))	Break ((50%)/(20%))	Break ((80%)/(50%))	Break ((100%)/(80%))
5×5×2	-1.4%	-0.1%	-8.0%	-2.4%
5×8×2	-2.3%	-5.0%	-5.6%	-1.2%
7×7×3	-2.3%	-3.1%	-5.1%	-3.3%
7×10×3	-1.8%	-4.0%	-5.0%	-5.0%
8×8×4	-2.0%	-5.3%	-6.8%	-1.6%
8×12×4	-2.9%	-4.8%	-3.8%	-2.2%
10×10×5	-2.4%	-1.9%	-5.9%	-1.8%
12×12×6	-1.4%	-3.2%	-3.0%	-3.3%
10×15×7	-2.5%	-4.7%	-5.7%	-2.6%
15×15×7	-4.8%	-3.0%	-4.1%	-0.9%

In Table 4, for most of the problem sizes, we observe a trend of higher decrease of minimum cost for Break((50%)/(20%)) as against other breakpoints comparisons indicated. We further show in Figure 2 that on the average across the breakpoint comparisons, Break((50%)/(20%)) tend to show the largest magnitude of decrease. This will imply a high likely hood of a firm reducing cost while meeting their demand obligations with breakpoints greater 50% of their demand. If a firm is constrained to operate below break 50%, then this computation will be necessary to observe their magnitude of cost decrease.



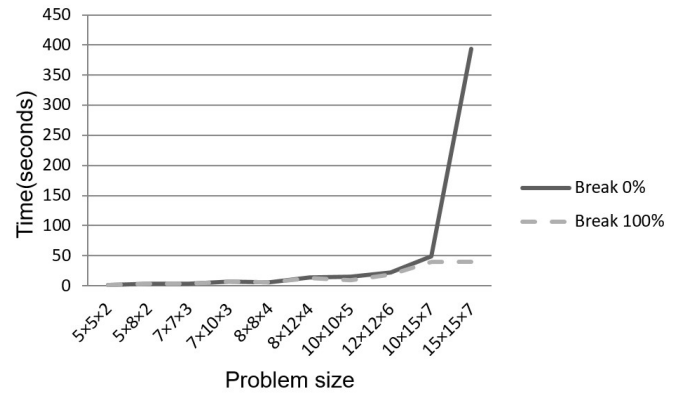
**Figure 2:** Minimum cost decrease against breakpoint intervals.

Under experimentation (B) we present Table 5 below to show the computation time in seconds across the breakpoints. As the breakpoint moves from Break 0% to Break 100%, the computation time shows a decreasing trend. This appears to be logical as the solution method is expected to compute easier solutions such as the FCSLTP when  $A_{ijr}$  (equals or approaches  $\max(D_r)$ ) or Break 100%, while the solution (decision) is expected to become more difficult when  $A_{ijr}$  (equals or approaches  $\min(D_r)$ ) or break 0%.

**Table 5:** Computation time (in seconds) across the breakpoints.

Problem size	Break 0%	Break 20%	Break 50%	Break 80%	Break 100%
5×5×2	1.499	1.431	1.388	1.365	1.356
5×8×2	3.880	4.087	3.907	3.652	3.660
7×7×3	4.086	4.179	4.281	4.243	4.068
7×10×3	6.994	7.063	7.369	7.401	6.706
8×8×4	6.581	6.774	6.590	6.200	6.119
8×12×4	14.749	14.097	14.292	13.867	13.432
10×10×5	15.084	14.711	11.975	10.279	9.878
12×12×6	22.903	24.014	24.953	21.550	19.039
10×15×7	49.444	47.300	42.002	40.853	39.768
15×15×7	393.378	76.094	72.688	49.733	40.592

In addition to the results obtained in Table 5, an increasing computation time trend is observable as the problem size increases across the breakpoints. The computation time at break 100% shows the lowest computation time across all problem sizes and breakpoints considered. The comparison of computation time as problem size increase between the breakpoint at minimum demand (break 0%) and at the maximum demand (break 100%) is displayed in figure 3 below. The computation time increase indicates the possibility of developing other solution methods such as heuristics that can provide good solutions quickly. This will be necessary for a firm constrained to make decisions under break 50%.



**Figure 3:** Computation time of Break 0% and Break 100% as the problem size increase.

**4. SUMMARY OF STEPS FOR MODEL APPLICATION**

1. In this section, we present a summary of the steps to apply the model.
2. Determine the input parameters such as the Facility location costs, route fixed charges etc.
3. Populate the SFCSLTP optimization model with the input parameters.
4. Determine the Breakpoints at (0%, 20%, 50 % and 100%) of demand.
5. Solve using the CPLEX dynamic solver at the breakpoints.
6. Interpret the solutions using the scenarios presented.
7. Determine the percentage decrease choosing two breakpoints close to each other.
8. Interpret the solutions based on the magnitude of the decrease obtained.
9. Use the interpretations to determine or negotiate which breakpoints will be desirable.

**5. CONCLUSION AND FUTURE DIRECTION**

**5.1 Conclusion**

A new optimization technique that integrates facility location decisions into a distribution problem known as the Step-Fixed Charge Solid Transportation Problem (SFCSTP) was considered. This problem was termed Step-Fixed Charge Solid Location and Transportation Problem (SFCSLTP) and solved using CPLEX optimization suite. We considered a particular step-fixed charge problem with two steps or fixed charges along the transportation route without loss of generalization. We discussed three scenarios of the possible occurrence of breakpoints. The first scenario occurring at the minimum demand point, the second scenario at the maximum demand, while the third scenario at a point in between the maximum and minimum demand requirements.

Results obtained confirmed the analytical prediction of a possible decrease in the minimum cost when the breakpoint tends towards the maximum demand point. The magnitude of decrease was however

largest when the breakpoint shifts beyond the average of the minimum and maximum demand, in the direction of the maximum demand. This could influence the decisions of managers when deciding policies that can make them operate at reduced cost when faced with more than one fixed charges on their shipping routes. This is especially when they cannot operate at the extreme scenarios of the breakpoints. The computation time also shows a decreasing trend when the breakpoint tends towards the maximum demand point. This possibly implies that the solution becomes quicker to solve when the breakpoint tends towards the maximum demand than when the breakpoint approaches the minimum.

## 5.2 Future direction

Possible extensions to our model could be modelling the SFCSLTP as a multi-objective programming problem that considers maximizing profit and minimizing the cost of the operating or planning personnel. In addition, the breakpoint which we have assumed to be constant for all the transportation routes can be considered stochastic. Lastly, another solution method can be developed to compare with the CLPEX solution in the areas of efficiency as a long solution time was observed with CPLEX with an increase in problem size.

## ACKNOWLEDGEMENT

The authors would like to acknowledge the University of Pretoria, Pretoria and the University of Ibadan, Nigeria for making their research facilities available to pursue this study.

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