



Copyright © 2010 American Scientific Publishers
All rights reserved
Printed in the United States of America

A Framework for Capturing Uncertainty of Group Decision-Making in the Context of the AHP/ANP

Lanndon Ocampo^{1,2a*}, Eppie Clark^{1b}

¹Department of Industrial Engineering, De La Salle University, 2401 Taft Avenue, 1004 Manila,
Philippines

²Department of Mechanical Engineering, University of San Carlos, Cebu City, 6000 Cebu, Philippines

^{a*}*don_leafriser@yahoo.com*

^b*eppie.clark@dlsu.edu.ph*

Abstract: Addressing uncertainty in the framework of the analytic hierarchy process (AHP) and its general form, the analytic network process (ANP), has been a dynamic field of research for the last four to five decades. Two directions of research emerged in this domain: the simulation approach and the fuzzy set theory (FST) approach. In this paper, we propose the integration of these two approaches in the context of AHP/ANP to elucidate group decision-making problems. FST is used to handle impreciseness of judgments of individual decision-maker while simulation is used to capture uncertainty brought about by the variability and randomness in aggregating decision-makers' judgments. These processes are applied at the level of the pairwise comparisons matrices in order to maintain the integrity of the general methodology of the AHP/ANP. The contribution of this work is on developing a methodology that addresses uncertainty both in individual and in group decision-making. The general framework and the detailed methodology are presented in this work. A simple case is used to show the computations.

Keywords: Analytic network process; Fuzzy set theory; Simulation; Uncertainty

1 INTRODUCTION

From its inception in the late 1970s, the analytic hierarchy process (AHP) and its general form, the analytic network process (ANP) have been very influential in the area of multi-criteria decision-making (MCDM). AHP/ANP is a theory of relative measurement proposed by Saaty [1] that systematically handles human objective and subjective judgment into a series of mathematical operations. Its widespread application especially in solving industry-related problems has been tremendous in the past four to five decades. Herva and Roca [2] surveyed literature on MCDM

applications and found out that AHP/ANP and outranking methods are commonly used in industry-related applications. Its applications include but not limited to computing product sustainability index [3], computing a time-dependent sustainability index [4], developing sustainability index for a manufacturing enterprise [5], developing multi-actor multi-criteria approach in complex sustainability project evaluation [6], evaluating industrial competitiveness [7], evaluating energy sources [8], developing an impact matrix and sustainability-cost benefit analysis [9], developing a reverse logistics model [10], conducting

sustainability assessment [11] especially in product and process design [12], etc.

Traditionally, eliciting judgment in pairwise comparisons in the context of the AHP was done with the help of the Saaty Fundamental Scale [1]. The scale denotes a 1 to 9-point judgment where 1 indicates indifference on a pair of elements with respect to a parent element and 2 up to 9 values represent the intensity of influence of the row element over the column element. One can think of these points as influence multipliers of row over column element. Saaty [13] posited that the pairwise comparisons matrix is a positive reciprocal matrix such that the 1-9 point scale assumes corresponding reciprocal values. The use of this scale forces a decision-maker to represent judgment in a single point estimate. When the scale is used, the presumption of preciseness and certainty of judgment is considered as the main drawback [14-15]. Saaty [16] noted this idea of uncertainty in the context of AHP/ANP. Zahedi [17] identified two possible sources of uncertainty in judgment. External sources concern the method or the environment in collecting preference data while internal sources refer to the “ambiguity and uncertainty which result from the limited amount of information available to the decision-maker and the level of his or her understanding of the problem” [14]. Paulson and Zahir [14] pointed out that these sources of uncertainty could possibly lead to rank reversals in the final ranking. This definitely affects the confidence of the decision makers with the results of the AHP/ANP. Saaty [16] and Vargas [18] made an attempt to resolve this issue by eliciting judgment in pairwise comparisons with random variables. Furthermore, Saaty and Vargas [19] analyzed the impact of uncertainty in the AHP using an interval approach of 100 simulated matrices. Later, this interval approach was utilized by Moreno-Jimenez and Vargas [20] in determining ranking of alternatives in the AHP. Zahir [21] developed an algorithm that solves a combinatorial problem by enumerating all possible combinations of uncertainties of the elements. This process requires significant amount of time in analyzing all possible combinations. Arbel and Vargas [22] introduced preference simulation and programming in obtaining priorities of a pairwise comparisons matrix. Paulson and Zahir [14] who are inspired with the approach of Saaty and Vargas [19] on interval simulation, studied how global priorities of AHP are affected by different decision levels and matrix sizes. Hauser and Tadikamalla [15] on the other hand introduced simulation of the AHP by introducing uniform and triangular probability distribution of judgment instead of a single point estimate traditionally practiced in AHP. Ahn [23]

who took a different view of the work of Hauser and Tadikamalla [15] discussed that their method of simulation is highly effective in group decision-making context where group members could not come up with a single point estimate.

Efforts on uncertainty propagation in the context of AHP/ANP for the last two decades have been inspired with the application of fuzzy set theory (FST). Various approaches have been introduced in integrating FST in AHP/ANP framework. These approaches include fuzzy logarithmic least squares method [24], a modified fuzzy logarithmic least squares method [25], geometric mean method [26], an extent analysis method [27], fuzzy least squares priority method [28], fuzzy preference programming method [29] and Lambda-Max method [30].

Uncertainty considerations using simulation and probabilistic approach could not handle individual impreciseness of decision-makers especially when judgments are elicited in a linguistic scale which is portrayed as the major advantage of FST. Although it was mentioned by Ahn's [23] short note that the work of Hauser and Tadikamalla [15] is suitable for group decision-making, there is no actual work that discusses this approach. On the other hand, FST approaches especially in group decision-making reduces the group judgments into a single judgment that assumes to describe the group decision. This approach falls short out of the context of uncertainty because the resulting group decision is assumed to be fully precise and certain. This paper attempts to establish a hybrid method that integrates simulation and FST approaches in the context of group decision-making in AHP/ANP. In this work, FST is used to handle judgmental uncertainty of individual decision-maker while simulation is intended to address the uncertainty of aggregated judgment of decision-makers. FST, as a way of handling uncertainty of decision-making in practical problems, has long been demonstrated in current literature [31-34]. On the other hand, probabilistic approaches were rarely used in solving real-life problems especially in group decision-making. Aside from the recommendation of Ahn [23] to use the simulation approach of Hauser and Tadikamalla [15] in group decision-making, Arunraj et al. [35] attempt to integrate FST and simulation in risk assessment but not in the context of ANP. The proposed hybrid approach eliminates the assumption of certainty in group decision and subscribes to a simulation framework where a single run describes a probable group decision. The contribution of this work lies in proposing a novel methodology that captures uncertainty and

randomness in group decision-making in the framework of the AHP/ANP.

2 ANALYTIC NETWORK PROCESS (ANP)

ANP is the general theory of relative measurement [36]. ANP decomposes a decision problem into a number of relevant components with their underlying interrelationships, thus forming a decision network. This flexibility of the ANP is one of the various reasons why the method encompasses other methods in the area of multi-attribute decision-making. The mainstream process of the ANP starts by carefully introducing dependence relations among decision components that show dominance of the elements of one component to the elements of another component. When the relationships agree with reality and observation, the process of pairwise comparisons becomes central to the process. Elements from the same component are compared pairwise with respect to the elements from another or the same component which those elements are being dominated. Local priorities of this pairwise comparisons matrix are accurately computed using the eigenvector method [1]. The eigenvalue problem to obtain the desired ratio-scale priority vector w of n elements is

$$Aw = \lambda_{\max}w \tag{1}$$

where A is the positive reciprocal pairwise comparisons matrix, λ_{\max} is the maximum or principal eigenvalue of the matrix A . λ_{\max} can be solved by rearranging equation (1) to form

$$A - \lambda_{\max}I = 0 \tag{2}$$

This eventually results to a polynomial of λ to the degree n and the largest root is the λ_{\max} . For consistent judgment, $\lambda_{\max} = n$, otherwise $\lambda_{\max} > n$. The measure of consistency of judgment is measured using the Consistency Index (CI) and Consistency Ratio (CR). The Consistency Index (CI) is a measure of the degree of consistency and is represented by

$$CI = \frac{\lambda_{\max} - n}{n - 1} \tag{3}$$

The consistency ratio (CR) is computed as

$$CR = \frac{CI}{RI} \tag{4}$$

where RI is the mean random consistency index obtained from large number of simulated matrices. $CR \leq 0.10$ is an acceptable degree of inconsistency.

After all local priority vectors are obtained from each pairwise comparisons matrix, the values are placed into the corresponding location of the

supermatrix. Global priority ratio scales or priorities can be computed based on the synthesizing principle of the supermatrix. The mathematical approach is by solving again a similar eigenvalue problem of equation (1). However, the numerical approach is done by raising the matrix to large powers where the transmission of influence along all possible paths defined in the decision structure is captured in the process [13]. The convergence of initial priorities (stochastic matrix) to an equilibrium value in the limit supermatrix provides a set of meaningful synthesized priorities from the underlying decision structure [37]. Saaty [13] guaranteed that as long as the decision network resembles a primitive irreducible supermatrix, the initial supermatrix will eventually converge to a limit supermatrix where each column is the global priority vector of all elements. The numerical approach of solving the limit supermatrix denoted by L is by normalizing columns and then raising the supermatrix to sufficiently large power [38] denoted by

$$\lim_{p \rightarrow \infty} \left(\frac{S}{\lambda_{\max}} \right)^p = \lim_{p \rightarrow \infty} (\bar{S})^p = L \tag{5}$$

3 FUZZY SET THEORY (FST)

Fuzzy set theory was introduced by Zadeh [39] as a mathematical way of handling imprecision and vagueness. Unlike traditional crisp set, FST takes on a membership function over a range of values. A fuzzy number can be represented by a fuzzy set $F = \{(x, u_F(x)), x \in R\}$ where x takes on any value on the real number line $R: -\infty < x < +\infty$ and $u_F(x)$ is a continuous mapping on the closed interval $[0, 1]$. There are several types of fuzzy numbers but the widely used one is the triangular fuzzy number (TFN). A TFN can be defined as a triple $A = (l, m, u)$ and the membership function $\mu_A(x)$ can be defined as

$$\mu_A(x) = \begin{cases} 0 & x < l \\ (x - l)/(m - l) & l \leq x \leq m \\ (u - 1)/(u - m) & m \leq x \leq u \\ 0 & x > u \end{cases} \tag{6}$$

and the representation of a TFN is

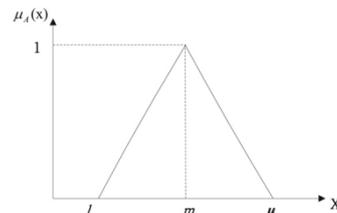


Fig. 1. A TFN $A = (l, m, u)$

Suppose two TFNs \tilde{A} and \tilde{B} are defined by (a_1, a_2, a_3) and (b_1, b_2, b_3) , respectively. The elementary operations of these two TFNs are as follows:

$$\tilde{A} + \tilde{B} = (a_1, a_2, a_3) + (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3) \quad (7)$$

$$\tilde{A} - \tilde{B} = (a_1, a_2, a_3) - (b_1, b_2, b_3) = (a_1 - b_1, a_2 - b_2, a_3 - b_3) \quad (8)$$

$$\tilde{A} \otimes \tilde{B} = (a_1, a_2, a_3) \times (b_1, b_2, b_3) = (a_1 b_1, a_2 b_2, a_3 b_3) \quad (9)$$

$$\tilde{A} \div \tilde{B} = (a_1, a_2, a_3) / (b_1, b_2, b_3) = (a_1 / b_3, a_2 / b_2, a_3 / b_1) \quad (10)$$

FST improves the capability of MCDM methods in handling complex and imprecise judgments. Most decision-makers can hardly elicit exact numerical values to represent opinions based on human judgment [40] because of various factors such as incomplete information to name one. More accurate judgment elicitation uses linguistic variables to represent judgment rather than numerical values [41]. Linguistic variables have values in the form of phrases or sentences in a natural language [42]. Expressing judgment in linguistic variable is a useful method in dealing with situations that are described in quantitative expressions [41][43]. Linguistic values can be represented by fuzzy numbers, and the TFN is commonly used [40].

4 PROPOSED METHODOLOGICAL FRAMEWORK

As discussed, several approaches have emerged on the integration of FST to the framework of AHP/ANP. The proposed procedure of integrating FST and simulation in the context of AHP/ANP that handles uncertainty in group decision-making is as follows:

1. Establish a decision model that appropriately describes the decision problem. The model could be a hierarchy or a network of decision components. The dependence relationships could be derived from a literature review of the subject, expert opinion or field surveys.
2. Prepare the set of pairwise comparisons matrices which correspond to the hierarchy or network in step 1. Note that the process is coherent with the framework of the AHP/ANP as proposed by Saaty [13].
3. Form a group of decision-makers who are experts in the decision problem. Provide the set of pairwise comparisons matrices to each expert. Experts elicit judgment in paired comparisons using the pre-defined linguistic variables with corresponding TFNs. The result

of this step is a set of pairwise comparisons matrices in linguistic variables or TFNs.

4. As discussed, there are various approaches in fuzzifying AHP/ANP. In this proposed method, the general framework of AHP/ANP is preserved. The idea is to defuzzify TFNs at the pairwise comparisons matrix level and then to proceed with the methodology of the ANP. Defuzzification is the process of appropriate selection of crisp element based on the output fuzzy set, which converts TFNs into crisp values [44]. In this study, the algorithm proposed by Opricovic and Tzeng [44] which is termed as the Converting the Fuzzy data into Crisp Scores (CFCS) method is used as it preserves the traditional approach of the AHP/ANP, unlike other popular methods such as in Mikhailov [29] which uses an optimization problem that maximizes the consistency index. This approach is highly regarded by Tseng in his works [45-46]. The results of CFCS are the crisp values of corresponding TFNs. The CFCS method is as follows:

Suppose a set of k decision-makers with $\tilde{w}_{ij}^k = (a_{1ij}^k, a_{2ij}^k, a_{3ij}^k)$ as the influence of i th element on j th element assessed by the k th evaluators.

Normalization:

$$xa_{1ij}^k = \frac{a_{1ij}^k - \min a_{1ij}^k}{\Delta_{\min}^{\max}} \quad (11)$$

$$xa_{2ij}^k = \frac{a_{2ij}^k - \min a_{2ij}^k}{\Delta_{\min}^{\max}} \quad (12)$$

$$xa_{3ij}^k = \frac{a_{3ij}^k - \min a_{3ij}^k}{\Delta_{\min}^{\max}} \quad (13)$$

where

$$\Delta_{\min}^{\max} = \max a_{3ij}^k - \min a_{1ij}^k.$$

Compute left ls and right rs normalized values

$$xls_{ij}^k = \frac{xa_{2ij}^k}{1 + xa_{2ij}^k - xa_{1ij}^k} \quad (14)$$

$$xrs_{ij}^k = \frac{xa_{3ij}^k}{1 + xa_{3ij}^k - xa_{2ij}^k} \quad (15)$$

Compute total normalized crisp value

$$x_{ij}^k = \frac{xls_{ij}^k(1 - xls_{ij}^k) + xrs_{ij}^k xrs_{ij}^k}{1 - xls_{ij}^k + xrs_{ij}^k} \quad (16)$$

Compute crisp values

$$w_{ij}^k = \min a_{1ij}^k + x_{ij}^k \Delta_{\min}^{\max} \quad (17)$$

where w_{ij}^k is the crisp value that represents the influence of element i on element j as described by the decision-maker k .

5. Compute the local eigenvector, λ_{\max} and C.R. values of the pairwise comparisons matrix using equations (5), (1) and (4), respectively. If C.R. < 0.10, the decision-maker must reconsider his/her judgment.
6. If C.R. < 0.10 in step 5 is satisfied, aggregating judgments of k number of decision-makers must be done. In this process, uncertainty arises due to the variability and randomness of crisp values eventually elicited by all decision-makers. Several works of Tseng [45-46] in this process recommended the simple averaging approach of aggregating judgment. This approach, however, does not guarantee that the variability of judgment is taken into consideration. With such gap, this work proposes a probabilistic approach in aggregating decision-makers' judgment. The probabilistic aggregated value w_{ij}^k can be computed using

$$\tilde{w}_{ij} = \bar{w}_{ij} \pm (1 - \alpha)p \quad (18)$$

where \tilde{w}_{ij} is an aggregated judgment of decision-makers from a normal distribution that represents the influence of row element on column element and \bar{w}_{ij} is the geometric mean of all judgment of decision-makers on the influence of element i on element j and is defined as

$$\bar{w}_{ij} = \sqrt[k]{w_{ij}^1 w_{ij}^2 w_{ij}^3 \dots w_{ij}^k} \quad (19)$$

$(1-\alpha)$ is the confidence level of the normal probability distribution and $p \in [0,1]$ is proportion of perturbation about the geometric mean. Paulson & Zahir [14] claimed that the value of p ranges from 2% to 20%. When α and p take on random values, a simulation process is described. Each simulation run describes a random chance of the aggregation uncertainty. When $p = 0$, then $\tilde{X}_{ij} = X_{ij}$. When p increases, the number of inconsistent judgment also increases and the more likely the rankings of text entry methods is changed compared to small values of p [15].

7. Using equations (5), (4) and (1), local eigenvectors are computed. These eigenvectors are placed in their appropriate locations in the supermatrix. Using equation (5), global priority vector can be computed. This vector is used to rank the elements on any given decision component. In each simulation run, corresponding global priority vector is computed. Thus, each run creates a distinct ranking of elements in a component. By conducting sufficiently large number of simulation runs (>100 runs), a matrix that describes the number of times an element is placed on any particular rank can be constructed. Normalizing columns provide the percentage of time that an element is placed on a given rank. These percentages can be considered as weights of the ordinal ranking positions. Hauser and Tadikamalla [15] defined the expected score of the i th element as follows

$$ES_i = \sum_{k=1}^n p_{i,k}(n + 1 - k) \quad \forall i \in [1, n] \quad (20)$$

Where ES_i is the expected score of the i th element, and $p_{i,k}$ is the percentage of the trials that the i th element has a rank k . After obtaining the expected scores, the expected weighted EW_i is defined as the normalized expected scores given in the following equation

$$EW_i = \frac{ES_i}{\sum_{k=1}^n ES_k} \quad \forall i \in [1, n] \quad (21)$$

where EW_i is considered a statistical weight from a scattered distribution of ranks to each element [15].

A sample illustrative computation is shown. Suppose a sample pairwise comparison matrix is shown in Table 1 and its corresponding matrix in TFNs is shown in Table 2. The equivalent TFNs were adopted from Tseng et al. [47] as shown in Table 3.

Table 1. Sample pairwise comparisons in linguistic scale

	Increased taxes	Environmenta l protection	Health and safety
Increased taxes		MO	MO
Environmental protection			EQ
Health and safety			

Table 2 Sample pairwise comparisons in TFNs

	Increased taxes	Environmental protection	Health and safety
Increased taxes	(1,1,1)	(5/2,3,7/2)	(5/2,3,7/2)
Environmental protection		(1,1,1)	1/2,1,3/2)
Health and safety			(1,1,1)

Table 3. Linguistic variables used in ANP (adopted from Tseng et al. [48])

Linguistic scale	Code	Triangular fuzzy scale	Triangular fuzzy reciprocal scale
Just equal		(1,1,1)	(1,1,1)
Equal importance	EQ	(1/2,1,3/2)	(2/3,1,2)
Moderate importance	MO	(5/2,3,7/2)	(2/7,1/3,2/5)
Strong importance	ST	(9/2,5,11/2)	(2/11,1/5,2/9)
Demonstrated importance	DE	(13/2,7,15/2)	(2/15,1/7,2/13)
Extreme importance	EX	(17/2,9,9)	(1/9,1/9,2/17)

Suppose that $i = 1, j = 2$ and $k = 1$, then

$$\tilde{w}_{12}^1 = \left(\frac{5}{2}, 3, \frac{7}{2}\right) = (2.500, 0.333, 3.500)$$

where

$$\Delta_{\min}^{\max} = \max a_{3ij}^1 - \min a_{1ij}^1 = \left(\frac{7}{2}\right) - \left(\frac{2}{7}\right) = 3.214$$

The normalized scores are

$$xa_{112}^1 = \frac{a_{112}^1 - \min a_{1ij}^1}{\Delta_{\min}^{\max}} = \frac{2.500 - 0.286}{3.214} = 0.689$$

$$xa_{212}^1 = \frac{a_{212}^1 - \min a_{1ij}^1}{\Delta_{\min}^{\max}} = \frac{3.000 - 0.286}{3.214} = 0.844$$

$$xa_{312}^1 = \frac{a_{312}^1 - \min a_{1ij}^1}{\Delta_{\min}^{\max}} = \frac{3.500 - 0.286}{3.214} = 1.000$$

Computing for the left and right normalized scores yields

$$xls_{12}^1 = \frac{xa_{212}^1}{1 + xa_{212}^1 - xa_{112}^1} = \frac{0.844}{1 + 0.844 - 0.689} = 0.731$$

$$xrs_{12}^1 = \frac{xa_{312}^1}{1 + xa_{312}^1 - xa_{212}^1} = \frac{1}{1 + 1 - 0.844} = 0.865$$

Computing for the total normalized crisp value yields

$$x_{12}^1 = \frac{xls_{12}^1(1 - xls_{12}^1) + xrs_{12}^1xrs_{12}^1}{1 - xls_{12}^1 + xrs_{12}^1} = \frac{0.731(1 - 0.731) + 0.865 * 0.865}{1 - 0.731 + 0.865} = 0.833$$

Computing for the crisp value shows

$$w_{12}^1 = \min a_{112}^1 + x_{12}^1 \Delta_{\min}^{\max} = 0.286 + 0.833(3.214) = 2.964$$

The corresponding pairwise comparisons matrix in crisp values is shown Table 4.

Table 4. Sample pairwise comparisons in crisp values

	Increased taxes	Environmental protection	Health and safety
Increased taxes	1.0000	2.9863	2.9863
Environmental protection	0.3349	1.0000	0.9723
Health and safety	0.3349	1.0285	1.0000

Following the aggregation process described in step 6, the aggregated pairwise comparisons matrix is presented in Table 5.

Table 5. Sample aggregated pairwise comparisons in crisp values

	Increased taxes	Environmental protection	Health and safety
Increased taxes	1.0000	3.3244	1.7114
Environmental protection	0.3008	1.0000	0.9873
Health and safety	0.5843	1.0128	1.0000

To summarize, the framework proposed in this study is shown in Fig. 2.

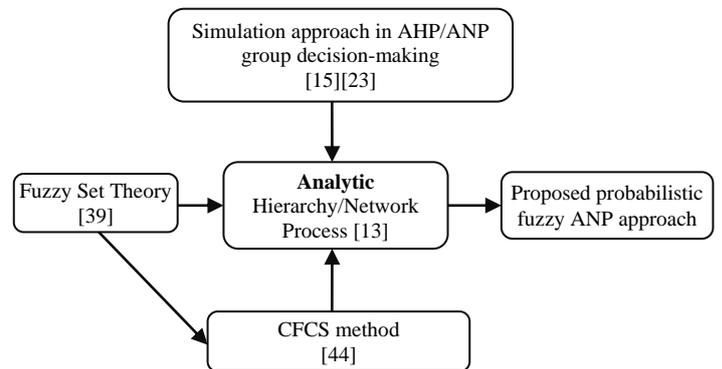


Fig. 2. Proposed methodological framework.

5 CONCLUSION

This work attempts to provide a methodological approach that holistically captures uncertainty in group decision-making. In this study, both judgmental uncertainty of individual decision-maker and the variability of judgment across decision-makers are fairly captured into a methodological structure which is described in this

paper. Unlike other approaches that largely deviate from the framework of the AHP/ANP, the proposed method preserved the general approach of the AHP/ANP. Fuzzy set theory is utilized to handle ambiguity of judgment of individual decision-maker while simulation in the context of normal probability distribution is used to address the variability and randomness of judgment of decision-makers. The contribution of this work is on developing a hybrid methodology that handles uncertainty effectively in the context of the AHP/ANP. Aside from the computational simplicity of the method, the proposed approach will also be helpful in handling group decision problems where uncertainty is extensive and where group members could hardly arrive at a consensus. Further work is required to empirically compare and examine the proposed method with previous methodologies in AHP/ANP group decision-making problems.

Acknowledgment

L. Ocampo recognizes the Ph.D. financial support of the Engineering Research and Development for Technology (ERDT) program of the Department of Science and Technology, Republic of the Philippines. The authors also recognize the insightful comments of two anonymous reviewers in improving the quality of this paper.

Reference

[1] T. L. Saaty, 1980. *The Analytic Hierarchy Process*, McGraw-Hill: New York.

[2] M. Herva, E. Roca, 2013. Review of combined approaches and multi-criteria analysis for corporate environmental evaluation, *Journal of Cleaner Production*, vol. 39, pp. 355-371.

[3] A. Gupta, R. Vangari, A.D. Jayal and I.S. Jawahir, 2011. Priority evaluation of product metrics for sustainable manufacturing, In: A. Bernard, editor. *Global Product Development*, Springer-Verlag Berlin Heidelberg, pp. 631-641.

[4] D. Krajnc and P. Glavic, 2005. A model for integrated assessment of sustainable development, *Resources, Conservation and Recycling*, vol. 43, no. 2, pp. 189-208.

[5] I.H. Garbie, 2011. Framework of manufacturing enterprises sustainability incorporating globalization issues, In: *Proceedings of the 41st International Conference on Computers and Industrial Engineering*, Los Angeles, CA USA.

[6] K. de Brucker, C. Macharis and A. Verbeke, 2013. Multi-criteria analysis and the resolution of sustainable development

dilemmas: a stakeholder management approach, *European Journal of Operational Research*, vol. 224, no. 1, pp. 122-131.

[7] S.B. Sirikrai and J.C.S. Tang, 2006. Industrial competitiveness analysis: using the analytic hierarchy process, *Journal of High Technology Management Research*, vol. 17, no. 1, pp. 71-83.

[8] A. Chatzimouratidis and P. Pilavachi, 2009. Technological, economic and sustainability evaluation of power plants using the Analytic Hierarchy Process, *Energy Policy*, vol. 37, no. 3, pp. 778-787.

[9] M.S. Chiacchio, 2011. Early impact assessment for sustainable development of enabling technologies, *Total Quality Management and Excellence*, vol. 39, no. 3, pp. 1-6.

[10] T.J. Barker and Z.B. Zabinsky, 2011. A multicriteria decision-making model for reverse logistics using analytical hierarchy process, *Omega*, vol. 39, no. 5, pp. 558-573.

[11] S. Seuring, 2013. A review of modeling approaches for sustainable supply chain management, *Decision Support Systems*, vol. 54, no. 4, pp. 1513-1520.

[12] N. Subramanian and R. Ramanathan, 2012. A review of applications of analytic hierarchy process in operations management, *International Journal of Production Economics*, vol. 138, no. 2, pp. 215-241.

[13] T.L. Saaty, 2001. *Decision making with dependence and feedback: The Analytic Network Process*, second ed., RWS Publications: Pittsburg, USA.

[14] D. Paulson and S. Zahir, 1995. Consequences of uncertainty in the analytic hierarchy process: A simulation approach, *European Journal of Operational Research*, vol. 87, no. 1, pp. 45-56.

[15] D. Hauser and P. Tadikamalla, 1996. The analytic hierarchy process in an uncertain environment: a simulation approach, *European Journal of Operational Research*, vol. 91, no. 1, pp. 27-37.

[16] T.L. Saaty, 1978. Modeling unstructured decision problems – the theory of analytic hierarchies, *Mathematics and Computers in Simulation*, vol. 20, no. 3, pp. 147-158.

[17] F. Zahedi, 1986. Group consensus function estimation when preferences are uncertain,

- Operations Research, vol. 34, no. 6, pp. 883-894.
- [18] L.G. Vargas, 1982. Reciprocal matrices with random coefficients, *Mathematical Modelling*, vol. 3, no. 1, pp. 69-81.
- [19] T.L. Saaty and L.G. Vargas, 1987. Uncertainty and rank order in the analytic hierarchy process, *European Journal of Operational Research*, vol. 32, no. 1, pp. 107-117.
- [20] J.M. Moreno-Jimenez and L.G. Vargas, 1993. A probabilistic study of preference structures in the analytic hierarchy process with interval judgments, *Mathematical and Computer Modelling*, vol. 17, no. 4-5, pp. 73-81.
- [21] M.S. Zahir, 1991. Incorporating the uncertainty of decision judgments in the analytic hierarchy process, *European Journal of Operational Research*, vol. 53, no. 2, pp. 206-216.
- [22] A. Arbel and L.G. Vargas, 1993. Preference simulation and preference programming: robustness issues in priority derivation, *European Journal of Operational Research*, vol. 69, no. 2, pp. 200-209.
- [23] B.S. Ahn, 2000. The analytic hierarchy process in an uncertain environment: A simulation approach by Hauser and Tadikamalla (1996), *European Journal of Operational Research*, vol. 124, no. 1, pp. 217-218.
- [24] P.J.M. Van Laarhoven and W. Pedrycz, 1983. A fuzzy extension of Saaty's priority theory, *Fuzzy Sets and Systems*, vol. 11, no. 1-3, pp. 199-227.
- [25] C. Boender, J.G. deGraan and F.A. Lootsma, 1989. Multi-criteria decision analysis with fuzzy pairwise comparisons, *Fuzzy Sets and Systems*, vol. 29, no. 2, pp. 133-143.
- [26] J.J. Buckley, 1985. Fuzzy hierarchical analysis, *Fuzzy Sets and Systems*, vol. 17, no. 3, pp. 233-247.
- [27] D. Y. Chang, 1996. Application of the extent analysis method on fuzzy AHP, *European Journal of Operational Research*, vol. 95, no. 3, pp. 649-655.
- [28] R. Xu, 2000. Fuzzy least-squares priority method in the analytic hierarchy process, *Fuzzy Sets and Systems*, vol. 112, no. 3, pp. 359-404.
- [29] L. Mikhailov, 2003. Deriving priorities from fuzzy pairwise comparison judgments, *Fuzzy Sets and Systems*, vol. 134, no. 3, pp. 365-385.
- [30] R. Csutora and J.J. Buckley, 2001. Fuzzy hierarchical analysis: the Lambda-Max method, *Fuzzy Sets and Systems*, vol. 120, no. 2, pp. 181-195.
- [31] M.L. Tseng, L. Divinagracia and R. Divinagracia, 2009. Evaluating firm's sustainable production indicators in uncertainty, *Computers & Industrial Engineering*, vol. 57, no. 4, pp. 1393-1403.
- [32] Y.M. Wang and K.S. Chin, 2008. A linear goal programming priority method for fuzzy analytic hierarchy process and its applications in new product screening, *International Journal of Approximate Reasoning*, vol. 49, no. 2, pp. 451-465.
- [33] M.L. Tseng, 2010. Implementation and performance evaluation using the fuzzy network balanced scorecard, *Computers and Education*, vol. 55, no. 1, pp. 188-201.
- [34] M.L. Tseng, R.J. Lin, Y.H. Lin, R.H. Chen and K. Tan, 2014. Close-loop or open hierarchical structures in green supply chain management under uncertainty, *Expert Systems with Applications*, vol. 41, no. 7, pp. 3250-3260.
- [35] N.S. Arunraj, S. Mandal and J. Maiti, 2013. Modeling uncertainty in risk assessment: An integrated approach with fuzzy set theory and Monte Carlo simulation, *Accident Analysis & Prevention*, vol. 55, pp. 242-255.
- [36] T.L. Saaty, 2008. The analytic hierarchy and analytic network measurement processes: applications to decisions under risk, *European Journal of Pure and Applied Mathematics*, vol. 1, no. 1, pp. 122-196.
- [37] M.A.B. Promentilla, T. Furuichi, K. Ishii and N. Tanikawa, 2008. A fuzzy analytic network process for multi-criteria evaluation of contaminated site remedial countermeasures, *Journal of Environmental Management*, vol. 88, no. 3, pp. 479-495.
- [38] P.F. Hsu and M.H. Kuo, 2011. Applying the ANP model for selecting the optimal full-service advertising agency, *International Journal of Operations Research*, vol. 8, no. 4, pp. 48-58.
- [39] L.A. Zadeh, 1965. Fuzzy set, *Information and Control*, vol. 18, no. 3, pp. 338-353.
- [40] M.L. Tseng, 2009. A causal and effect decision-making model of service quality expectation using grey-fuzzy DEMATEL approach, *Expert Systems with Applications*, vol. 36, no. 4, pp. 7738-7748.
- [41] R.C. Wang and S.J. Chuu, 2004. Group decision-making using a fuzzy linguistic

- approach for evaluating the flexibility in a manufacturing system, *European Journal of Operational Research*, vol. 154, no. 3, pp. 563–572.
- [42] C. Von Altrock, 1996. Practical Fuzzy-Logic Design, *The Computer Applications Journal*, Circuit Cellar INK., vol. 75, pp. 1-5.
- [43] U. Asan, C. Erhan Bozdag and S. Polat, 2004. A fuzzy approach to qualitative cross impact analysis, *Omega – International Journal of Management Science*, vol. 32, no. 6, pp. 443–458.
- [44] S. Opricovic and G.H. Tzeng, 2003. Defuzzification within a multi-criteria decision model, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, vol. 11, no. 5, pp. 635–652.
- [45] M.L. Tseng, 2011. Green supply chain management with linguistic preferences and incomplete information, *Applied Soft Computing*, vol. 11, no. 8, pp. 4894-4903.
- [46] M.L. Tseng and A.S.F. Chiu, 2013. Evaluating firm's green supply chain management in linguistic preference, *Journal of Cleaner Production*, vol. 40, no. 22-31.
- [47] M.L. Tseng, Y.H. Lin, A.S.F. Chiu and J.C.H. Liao, 2008. Using FANP approach on selection of competitive priorities based on cleaner production implementation: a case study in PCB manufacturer, Taiwan, *Clean Technologies and Environmental Policy*, vol. 10, no. 1, pp. 17-29.

