Stochastic Analysis of a Cold Standby System with Priority to Preventive Maintenance over Repair

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Abstract: A stochastic model is developed for a two-unit cold standby system under the aspect of priority to preventive maintenance over repair. The units are identical having two modes- operative and complete failure. There is a single server who visits the system immediately to carry out the repair activities. Server conducts preventive maintenance of the operative unit after a pre-specific time 't'. However, repair of the unit is done by the server at its complete failure. Priority is given to maintenance of one unit over repair of the other unit. The random variables associated with failure time, the rate by which unit undergoes for preventive maintenance and repair time are statistically independent. The failure time and time by which unit goes for preventive maintenance follow exponential distribution while the distributions for maintenance and repair times are taken as arbitrary with different probability density functions. Several measures of system effectiveness are obtained using semi-Markov process and regenerative point technique. Graphs are drawn to depict the behavior of MTSF, availability and profit function giving arbitrary values to various parameters and costs.

Keywords: cold standby system; maximum operation time; preventive maintenance; repair; priority and stochastic analysis

1 INTRODUCTION

The stochastic models of cold standby systems have extensively been studied by the researchers including Nakagawa (1975), Goel and Sharma (1989) and Dhillon (1992) due to their wide applications in the industries. It is a known fact that most of the operating systems are vulnerable to damage caused by wear out and other unforeseen reasons. Therefore, preventive maintenance of such systems become necessary after a pre-specified operation time in order to maintain their life and efficiency for a considerable period. Chelbi et al. (2008) suggested optimal inspection and preventive maintenance policy for randomly failing systems. Also, Malik (2013) introduced the concept of preventive maintenance while analyzing a computer system with cold standby redundancy. Further, system availability may be improved by giving priority in repair discipline of one unit over the other. Chhillar and Malik (2013) determined reliability measures of a stand by

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shock model giving priority to maintenance over repair.

In view of these practical situations in mind, here a two-unit cold standby system of identical units is analyzed stochastically in detail giving priority to preventive maintenance over repair. The unit has two modes- operative and complete failure. There is a single server who visits the system immediately for conducting maintenance and repair. Server conducts preventive maintenance of the unit after a maximum operation time't'. However, repair of the unit is done at its complete failure. Priority is given to maintenance of one unit over repair of the other unit. The random variables associated with failure time, completion of maximum operation time, preventive maintenance and repair times are statistically independent. The failure time of the unit and the time by which unit undergoes for preventive maintenance follow negative exponential distribution while the distributions for maintenance and repair times are taken as arbitrary. Several measures of system effectiveness such as transition probabilities mean sojourn times, mean time to system failure (MTSF), availability, busy period of the server due to maintenance and repair, expected number of maintenances and repairs of the unit and profit function are obtained using semi-Markov process and regenerative point technique. Graphs are drawn to depict the behavior of MTSF, availability and profit function for particular values of various parameters and costs.

2 RESEARCH APPROACH

The system model has been analyzed stochastically in detail using the following approach:

Semi-Markov Process

A continuous-time stochastic process is called a semi-Markov process if transition from one state to another is governed by the transition probabilities of a Markov process and in which time interval between two successive transition is a random variable whose distribution may depend upon the state from which transition takes place as well as on the state to which next transitions take place.

Mathematically,

In the above, assume that the process is time homogeneous, i.e.

$$\Pr\{X_{n+1}=j, t_{n+1}-t_n \le t \mid X_n = i\} = Q_{ij}(t), i, j \in s,$$

is independent of n, then there exit limiting transition probabilities.

$$p_{ij} = \lim_{t\to\infty} Q_{ij}(t) = \Pr\{X_{n+1} = j \mid X_n = i\}$$

Then $\{X_n, n = 0,1,2,3,...\}$ constitute a Markov Chain with state space E and transition probability matrix (t.p.m) $P = [p_{ij}]$

The continuous parameter stochastic process Y(t) with state space E defined by

$$Y(t) = X_n, t_n < t < t_{n+1}$$

is called a semi-Markov process. The Markov chain X_n is said to be an embedded Markov chain of the semi-Markov process.

3 NOTATIONS

E_0 : Set of regenerative states

O/Cs : The unit is operative/cold stand by

α_0 : The rate by which unit undergoes for preventive maintenance (called maximum constant rate of operation time)

λ : Constant failure rate of the unit.

f(t)/F(t) : pdf /cdf of preventive maintenance time

g(t)/G(t) : pdf /cdf of repair time of a failed unit

P_m/WP_m : The unit is under preventive maintenance/waiting for preventive maintenance

PM/FUR: The unit is continuously under preventive maintenance/under repair from previous state

FU_r/Fw_r : The failed unit under repair/waiting for repair

m_{ij} : The unconditional mean time taken by the system to transit from any regenerative state S_i when it (time) is counted from epoch of entrance in to that state S_j . Mathematically it can be written as

$$m_{ij} = \int_0^\infty t d[Q_{ij}(t)] = -q_{ij}^{*'}(0)$$

μ_i : The mean Sojourn time in state S_i which is given by

$$\mu_i = E(t) = \int_0^\infty P(T > t) dt = \sum_j m_{ij} ,$$

where T denotes the time to system failure

$W_i(t)$: Probability that the server is busy in the state S_i up to time 't' without making any transition to any other regenerative state or returning to the same state via one or more regenerative states.

⊗/⊕ : Symbol for Laplace Stieltjes convolution/Laplace convolution

~/* : Symbol for Laplace Steltjes transform (LST)/ Laplace transform (LT)

The possible transition states are shown in Fig. 1.

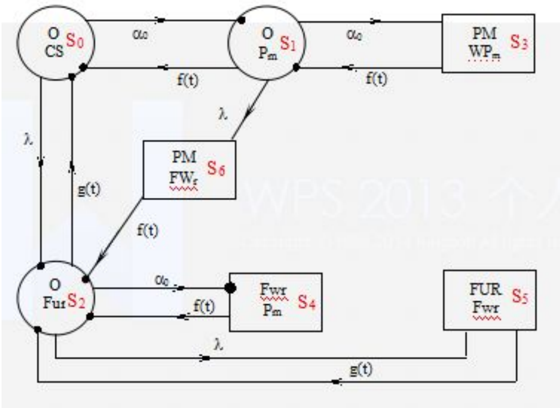

Fig. 1 State Transition Diagram

4 TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Simple probabilistic considerations yield the following expressions for non-zero elements

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt \quad (1)$$

$$p_{01} = \frac{\alpha_0}{\alpha_0+\lambda},$$

$$p_{02} = \frac{\lambda}{\alpha_0+\lambda}, \; p_{10} = f^*(\alpha_0+\lambda),$$

$$p_{13} = p_{11.3} = \frac{\alpha_0}{\alpha_0+\lambda}[1-f^*(\alpha_0+\lambda)]$$

$$p_{16} = \frac{\lambda}{\alpha_0+\lambda}[1-f^*(\alpha_0+\lambda)],$$

$$p_{20} = g^*(\alpha_0+\lambda),$$

$$p_{25} = p_{22.5} = \frac{\lambda}{\alpha_0+\lambda}[1-g^*(\alpha_0+\lambda)]$$

$$p_{24} = \frac{\alpha_0}{\alpha_0+\lambda}[1-g^*(\alpha_0+\lambda)],$$

$$p_{31} = p_{52} = p_{62} = 1 \quad (2)$$

It can be easily verified that

$$p_{01}+p_{02} = p_{10}+p_{13}+p_{16} = p_{20}+p_{24}+p_{25} = 1 \quad (3)$$

The mean sojourn times (μ_i) is the state S_i are

$$\mu_0 = \frac{1}{\alpha_0+\lambda},$$

$$\mu_1 = \frac{1}{\alpha_0+\lambda}[1-f^*(\alpha_0+\lambda)],$$

$$\mu_2 = \frac{1}{\alpha_0+\lambda}[1-g^*(\alpha_0+\lambda)],$$

$$\mu_1' = \frac{1}{\alpha} \text{ and } \mu_2' = \frac{\theta+\lambda}{\theta(\theta+\lambda+\alpha_0)} \quad (4)$$

5 RELIABILITY AND MEAN TIME TO SYSTEM FAILURE (MTSF)

Let $\phi_i(t)$ be the cdf of first passage time from the regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$;

$$\phi_i(t) = \sum_j Q_{i,j}(t) \otimes \phi_j(t) + \sum_k Q_{i,k}(t) \quad (5)$$

where S_j is an un-failed regenerative state to which the given regenerative state 'S_i' can transit and S_k is failed state to which the state S_i can transit directly. Taking LT of relation (5) and solving for $\tilde{\varphi}_0(s)$. We have

$$R^*(s) = \frac{1-\tilde{\phi}_0(s)}{s} \quad (6)$$

The reliability of the system model can be obtained by taking Laplace inverse transform of (6). The mean time to system failure (MTSF) is given by

$$MTSF = \lim_{s\to 0}\frac{1-\tilde{\phi}_0(s)}{s} = \frac{\mu_0+\mu_1 p_{01}+\mu_2 p_{02}}{1-p_{01}p_{10}-p_{02}p_{20}} \quad (7)$$

6 STEADY STATE AVAILABILITY

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state S_i at t=0. The recursive relations for $A_i(t)$ are given as

$$A_i(t) = M_i(t) + \sum_j q_{i,j}^{(n)}(t) \oplus A_j(t) \quad (8)$$

Where S_j is any successive regenerative state to which the regenerative state S_i can transit through n transitions. $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state, we have

$$M_0(t) = e^{-(\alpha_0+\lambda)t}, \; M_1(t) = e^{-(\alpha_0+\lambda)t}\overline{F(t)},$$

$$M_2(t) = e^{-(\alpha_0+\lambda)t}\overline{G(t)} \quad (9)$$

Taking LT of relation (9) and solving for $A_0^*(s)$. The steady state availability is given by

$$A_0(\infty) = \lim_{s \to 0} sA_0^*(s) = \frac{N}{D} \tag{10}$$

where

$N = \mu_0(1-P_{11.3})\{(1-P_{22.5})-P_{24}P_{42}\} + \mu_1 P_{01}\{(1-P_{22.5}-P_{2}$

$+\mu_2\{P_{01}P_{12.6}+P_{02}(1-P_{11.3})\}$

$D = \mu_0(1-P_{11.3})\{(1-P_{22.5})-P_{24}P_{42}\} + \mu_1' P_{01}\{(1-P_{22.5}-P_{24}P_{42}\}$

$+\mu_2'\{P_{01}P_{12.6}+P_{02}(1-P_{11.3})\} + \mu_4\{p_{01}p_{12.6}p_{24}+p_{24}p_{02}(1-p_{11.3}$

7 BUSY PERIOD ANALYSIS FOR SERVER

Let $B_i^p(t)$ and $B_i^R(t)$ be the probability that the server is busy in preventive maintenance and repair of the unit at an instant 't' given that system entered state S_i at t=0. The recursive relations for $B_i^p(t)$ and $B_i^R(t)$ are as follows:

$$B_i^p(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) \oplus B_j^p(t)$$

$$B_i^R(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) \oplus B_j^R(t) \tag{11}$$

Where S_j is any successive regenerative state to which the regenerative state S_i can transit through n transitions. $W_i(t)$ be the probability that the server is busy in state S_i due to preventive maintenance up to time 't' without making any transition to any other regenerative state or returning to the same via one or more non-regenerative state and so

$$W_1(t) = \{e^{-(\alpha_0+\lambda)t} + (\alpha_0 e^{-(\alpha_0+\lambda)t} \oplus 1) + (\lambda e^{-(\alpha_0+\lambda)t} \oplus 1)\}\overline{F(t)}$$

$W_2(t) = e^{-(\alpha_0+\lambda)t}\overline{G(t)}$ and $W_4(t) = \overline{F(t)}$

Taking Laplace Transformation of above relations (11). Solving for $B_0^{*p}(t)$ and $B_0^{*R}(t)$, the time for which server is busy due to preventive maintenance and repair respectively is given by

$$B_0^p(t) = \lim_{s \to 0} sB_0^{*p}(t) = \frac{N_1^p}{D} \text{ and}$$

$$B_0^R(t) = \lim_{s \to 0} sB_0^{*R}(t) = \frac{N_2^R}{D} \tag{12}$$

where

$N_1^p(t) = W_1^*\left(p_{01}(1-p_{22.5}) + p_{24}p_{42}\right) + W_4^*\left(p_{01}p_{12.6}p_{24} + p_{02}p_{24}(1-p_{11.3})\right)$

$N_2^R(t) = W_2^*\{p_{01}p_{12.6} + p_{02}(1-p_{11.3})\}$

and D has already mentioned.

8 EXPECTED NUMBER OF REPAIRS AND PREVENTIVE MAINTENANCES OF THE UNIT

Let $R_i^R(t)$ and $R_i^P(t)$ be the expected number of repairs and preventive maintenances of unit by the server in (0,t] given that the system entered the regenerative state S_i at t=0. The recursive relations for $R_i^R(t)$ and $R_i^P(t)$ are given as

$$R_i^R(t) = \sum_j Q_{i,j}^{(n)}(t) \oplus [\delta_j + R_j^R(t)] \quad \text{and}$$

$$R_i^S(t) = \sum_j Q_{i,j}^{(n)}(t) \oplus [\delta_j + R_j^S(t)] \tag{13}$$

where S_j is any regenerative state to which the given regenerative state S_i transits and δ_j =1, if S_j is the regenerative state where the server does the job afresh, otherwise δ_j = 0. Taking LT of relations (13) and, solving for $\tilde{R}_0^R(t)$ and $\tilde{R}_0^P(t)$. The expected no of repairs and preventive maintenances per unit time are respectively given by

$$R_0^R(\infty) = \lim_{s \to 0} s\tilde{R}_0^R(s) = \frac{N_3^R}{D} \quad \text{and}$$

$$R_0^P(\infty) = \lim_{s \to 0} s\tilde{R}_0^P(s) = \frac{N_4^P}{D} \tag{14}$$

where

$N_3^R = p_{01}(1-p_{22.5}) + p_{24}p_{42}\{-p_{01}(1-p_{12.6}) + p_{02}(1-p_{11.3})\}$

$N_4^P = (p_{20}+p_{22.5}+p_{24})\{p_{01}p_{12.6}+p_{02}(1-p_{11.3})\} + \{p_{01}p_{12.6}+p_{02}(1-p_{11.3})\}$

and D has already defined.

9 PROFIT ANALYSIS

The profit incurred to the system model in steady state can be obtained as

$$P = K_0A_0 - K_1B_0^P - K_2B_0^R - K_3R_0^R - K_4R_0^P - K_5 \tag{15}$$

K_0= Revenue per unit up-time of the system

K_1= Cost per unit time for which server is busy due preventive maintenance

K_2= Cost per unit time for which server is busy due to repair

K_3= Cost per unit time repair

K_4= Cost per unit time preventive maintenance

K_5= Total installation cost of the system

10 PARTICULAR CASE

Let us take $g(t) = \theta e^{-\theta t}$ and $f(t) = \alpha e^{-\alpha t}$, then the following results are obtained:

MTSF=

$$\frac{(\alpha+\lambda+\alpha_0)(\theta+\lambda+\alpha_0)+\alpha_0(\theta+\lambda+\alpha_0)+\lambda(\alpha+\lambda+\alpha_0)}{(\lambda+\alpha_0)(\alpha+\lambda+\alpha_0)(\theta+\lambda+\alpha_0)-\alpha\alpha_0(\theta+\lambda+\alpha_0)-\lambda\theta(\alpha+\lambda+\alpha_0)} \quad (16)$$

Availability A_0=

$$\frac{\alpha\theta(\theta+\alpha_0)(\lambda+\alpha_0+\alpha)}{\alpha\theta(\lambda+\alpha)+\alpha_0\theta^2(\lambda+\alpha+\alpha_0)+\alpha\lambda(\theta+\lambda)(\alpha+\lambda+\alpha_0)+\alpha_0\theta(\lambda\alpha_0+(\lambda+\alpha_0)(\alpha+\lambda))} \quad (17)$$

Busy period for preventive maintenance

$$B_0^p=\frac{\alpha_0\theta(\alpha+\lambda+\alpha_0)(2\lambda+\theta+2\alpha_0)}{\alpha\theta(\lambda+\alpha)+\alpha_0\theta^2(\lambda+\alpha+\alpha_0)+\alpha\lambda(\theta+\lambda)(\alpha+\lambda+\alpha_0)+\alpha_0\theta(\lambda\alpha_0+(\lambda+\alpha_0)(\alpha+\lambda))} \quad (18)$$

Busy period for Repair

$$B_0^R=\frac{\alpha\lambda(\alpha+\lambda+\alpha_0)(\lambda+\theta)}{\alpha\theta(\lambda+\alpha)+\alpha_0\theta^2(\lambda+\alpha+\alpha_0)+\alpha\lambda(\theta+\lambda)(\alpha+\lambda+\alpha_0)+\alpha_0\theta(\lambda\alpha_0+(\lambda+\alpha_0)(\alpha+\lambda))} \quad (19)$$

Expected Number of visits for preventive maintenance

$$R_0^p=\frac{\alpha\theta\{\alpha_0(\theta+\alpha_0)(\lambda+\theta+\alpha_0)+\alpha_0(\lambda-\alpha_0)(\theta+\lambda+\alpha_0)\}}{\alpha\theta(\lambda+\alpha)+\alpha_0\theta^2(\lambda+\alpha+\alpha_0)+\alpha\lambda(\theta+\lambda)(\alpha+\lambda+\alpha_0)+\alpha_0\theta(\lambda\alpha_0+(\lambda+\alpha_0)(\alpha+\lambda))} \quad (20)$$

Expected Number of visits for repair

$$R_0^R=\frac{\lambda\alpha\theta(\alpha+\lambda+\alpha_0)(\lambda+\theta+\alpha_0)}{\alpha\theta(\lambda+\alpha)+\alpha_0\theta^2(\lambda+\alpha+\alpha_0)+\alpha\lambda(\theta+\lambda)(\alpha+\lambda+\alpha_0+\alpha_0\theta(\lambda\alpha_0+(\lambda+\alpha_0)(\alpha+\lambda))} \quad (21)$$

11 CONCLUSION

The graphs for mean time to system failure, availability and profit function have been drawn with respect to preventive maintenance rate (α) giving particular values to the parameters and costs as shown respectively in Figure 2, 3 and 4. It is observed that the value of these reliability measures go on increasing with the increase of preventive maintenance and repair rates. But their values decline with the increase of the rate (α_0) by which the unit undergoes for preventive maintenance as well as by increasing failure rate of the unit. However, profit of the system has been evaluated after reducing the installation cost of the system which has generally been ignored by the researchers. It is interesting to note that the system model will always be in loss for α_0=7, $\lambda=.01$ *and* $\theta=2.5$.

Finally, it is concluded that a cold standby system in which priority is given to preventive maintenance over repair can be made more profitable to use either by increasing the preventive maintenance and repair rates or by reducing the rate by which system under goes for preventive maintenance.

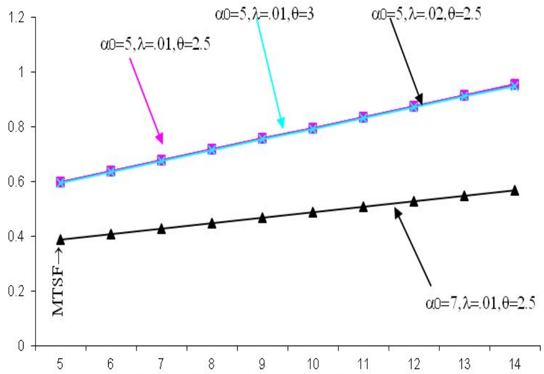

Fig. 2: Premaintive maintenance rate(α) →

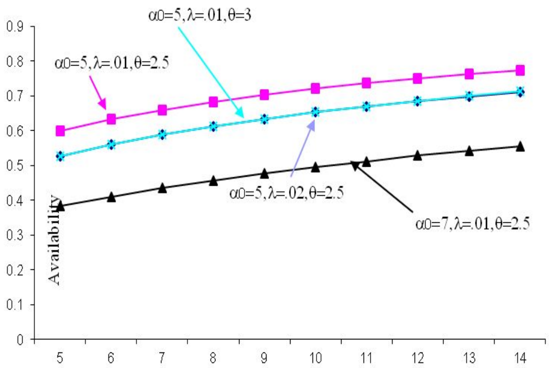

Fig. 3: Premaintive maintenance rate(α) →

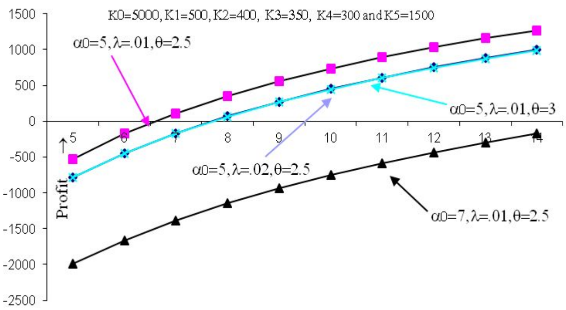

Fig. 4: Premaintive maintenance rate(α)→

12 SIGNIFICANCE OF THE WORK

The application of the present study can be visualized in a water supply system of two identical electric pumps – one is initially working and the other is kept as spare in cold standby. On the other hand, the results can be used theoretically by those researchers working in the areas of Reliability Engineering and Mathematical Modeling.

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