Lot Size Formulation Minimizing Makespan with Transactional and Movement Times

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Abstract: This paper proposes a lot size formulation with the objective of minimizing manufacturing makespan. A simple manufacturing system is considered with single product that is processed through pre-defined task sequence of known tasks, transactional and material handling times. The lot size problem yields a nonlinear optimization model whose solution satisfies the Kuhn-Tucker necessary and sufficient conditions. Results show that lot size is a function of the demand, the transactional activities and the length of the material handling times at the constraint task, and the sum of unconstrained task times. When transactional and material-handling times are considered, optimal lot size is independent of the longest task time but dependent on the sum of the processing times of the unconstrained tasks. The main contribution of this paper is the proposed method of minimizing makespan without spending resources in improving the constraint task but instead, considers developing an optimal lot size. This paper poses relevant applications especially for high-volume manufacturing systems.
Keywords: lot size; nonlinear optimization; makespan

1 INTRODUCTION

Much has been written on lot size problem ranging from deterministic to probabilistic and stochastic models. Due to its importance in almost all manufacturing environment, it has been one of the central foci of discussion in various scientific journals on industrial engineering. The classical work of Khabibullin [1] presented a mathematical formulation of the optimal batch size problem for manufacturing operations in the framework of production planning and control system for a parts manufacturing shop of a machine-building plant. He proposed a generalized descent algorithm for the solution of the problem. Hamada [2] described a Bayesian sequential batch-size decision model to minimize expected total completion time on single machine using dynamic programming. Van Vyve [3] developed a third order algorithm $O(n^3)$ for the single-item lot-sizing problem with constant batch size backlogging and later on proposed linear programming extended formulations of same problem. Bar-noy et al. [4] also studied throughput maximization of real-time scheduling with batching. Halim [5] presented a batch scheduling problem for

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a single machine that processes a number of parts for a single item. Halim [5] developed a program which relates batch size quantity, setup time and processing time to minimize actual flow time of all parts. Li et al. [6] studied makespan estimation of multi-purpose batch process in a traditional job-shop type of production system using a back-propagation network (BPN) combined with genetic algorithm (GA). Rao et al. [7] formulated an economic lot sizing scheduling model with fuzzy inventory costs and fuzzy objective function using fuzzy genetic algorithm. Parsa et al. [8] studied minimizing makespan of single batch processing machine using branch and price algorithm with column generation technique. Xu et al. [9] studied minimizing makespan of batch processing machine using mixed integer programming and ant colony optimization. The proposed algorithms were found to be robust with good solution quality and efficient computing time. Ramezanian et al. [10] explored multi-product and multi-period lot sizing and scheduling models with preventive maintenance using mixed-integer programming. A conceptual approach introduced by lean manufacturing enthusiasts is the one-piece flow approach of the renowned Toyota Production System (TPS) [11]. The approach is argued to minimize inventory costs, guard associated costs related to manufacturing line breakdown, improve productivity and enhance quality. A process model was introduced by Hanenkamp [11] that integrates shop floor management concepts into a cohesive framework that supports the objectives of lean manufacturing. However, such one-piece flow approach becomes irrelevant when necessary transactional and movement times become unavoidable in the shop floor. In a highly standardized, high-volume manufacturing system and similar systems, deciding lot size is crucial in determining demand completion time with consequent impact on production schedules, resource utilization, etc.

In this paper, a single-item continuous manufacturing system is being considered. A single-item processed in lot of unknown size runs through N tasks sequenced according to task $j = 1,2,3,\dots,N$ where each task consists of a single machine to complete a production cycle. Given a manufacturing lead time, the system should produce the quantity demand, required by customers. In completing the quantity demand D, each task is required to have transactional activities necessary for monitoring production schedules, checking lot information, updating the information system, and other activities pertinent to the lot at hand. The transactional time, in our manufacturing model for a specific task is unique and is known at present. Further, each task has known task time to complete a unit of output denoted by t_j, independent from each other. Other characteristics of the model includes the material-handling time, H_j performed from task $j \rightarrow j+1 \leq N$, where H_j is also unique at every stage of the production process. Since the system is continuous, the manufacturing lead time should equal to the gross available production time. The due date to complete demand D is fixed and the production schedule is determined through backward scheduling. A quantity lot should be produced k times in order to complete D. The actual time to complete D is defined as the makespan which should be less than manufacturing lead time so as to satisfy due date. This paper is focused on minimizing the makespan, an essential metric for most manufacturing systems and a critical parameter especially for high volume production. Minimal makespan makes the system efficient to produce customer demands faster. Several literatures draw interest on makespan with different approaches [6, 8-9].

This paper, on the other hand, presents minimizing manufacturing makespan as a function of lot size. One of the practical approaches to minimize makespan is to focus on improving the constraint task (longest task time) in a manufacturing process since constraint task dictates the rate of the system [12]. Nevertheless, in this approach, it takes a lot of cost, effort and extra amount of resources to increase the speed of the constraint task. In many cases, it usually ends up delimiting productivity. In our study, makespan was minimized by taking the optimal lot size having transactional and material-handling time on hand.

2 PROBLEM FORMULATION AND MODELLING

The following summarizes the notations used in this paper.

Let,

t_j processing time of task $j = 1,2,3\dots,N$ to complete one unit of output, (unit of time per unit of output)

P_j pre-processing transactional time required for preparatory activities before start of a task j (unit of time)

H_j material-handling time from task j to $j+1$ (unit of time)

$C_{[k]}$ completion time of a lot k (unit of time)

D demand placed by customer with a specified due date (unit of output)

Q^* optimal lot-size quantity (unit of output)

$M(Q)$ actual makespan (unit of time)

In this paper, we follow a just-in-time (JIT) approach which basically focuses on backward scheduling technique [5]. JIT is a sound controlling device for scheduling decisions for several manufacturing systems. In our model, we assume that after the first lot is processed on the first task, the second lot immediately follows then the succeeding lots. Therefore a particular lot k at task j cannot be processed at task $j+1$ until the lot $k-1$ is done at task $j+1$. This is practically known as the Johnson's scheduling rule. Further, we assume that delays and machine downtimes are negligible compared to the total task time. Fig. 1 presents the following,

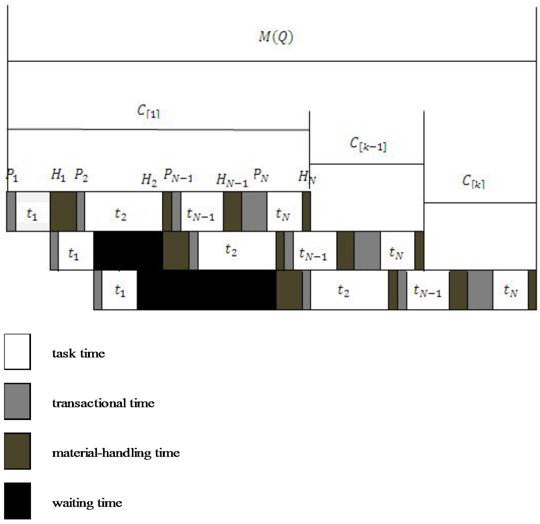

Fig. 1 The total makespan $M(Q)$

Each task in the manufacturing system has a respective $(P_i + Qt_i)$ time for a lot size Q. In this case, irrespective of Q, P_i is a constant term and the term $\sum_{m=1}^{k}\sum_{i=1}^{n} P_{im}$ may increase significantly for small values of Q such that $kQ = D$. Adding up the material-handling time, each task now can have $\{(P_i + Qt_i) + H_i\}$ where H_i is the time to move Q from task j to $j+1$ and is constant regardless of Q. Now, to complete the first lot Q for demand D, the total manufacturing time is,

$$C_{[1]} = \sum_{i=1}^{n}\{(P_i + Qt_i) + H_i\} \tag{1}$$

The time for the remaining lots $(k-1)Q$ can be explained as follows. For continuous manufacturing system, the time it takes to produce another lot is equal to the longest task time (constraint task) assuming considerably insignificant waste time. For our model, each task has properties P_i and H_i, hence the completion time of succeeding lots must also possess P_i and H_i. Now, the number of lots to complete D is equal to

$$k = \frac{D}{Q} \tag{2}$$

and the number of lots after the first lot has been completed is,

$$k - 1 = \frac{D}{Q} - 1 \tag{3}$$

We can now formulate the completion time of $k-1$ lots as the following,

$$C_{[k-1]} = \left(\frac{D}{Q} - 1\right)\max\{(P_j + Qt_j) + H_{j-1}\}_{j=1}^{N} \tag{4}$$

Hence, the manufacturing makespan can be written as follows,

$$M(Q) = \sum_{j=1}^{N}\{(P_j + Qt_j) + H_j\} + \left(\frac{D}{Q} - 1\right)\max\{(P_j + Qt_j) + H_{j-1}\}_{j=1}^{N} \tag{5}$$

The optimization model of minimizing manufacturing makespan can be described as,

$$\min_Q M(Q) = \sum_{j=1}^{N}\{(P_j + Qt_j) + H_j\} + \left(\frac{D}{Q} - 1\right)\max\{(P_j + Qt_j) + H_{j-1}\}_{j=1}^{N} \tag{6}$$

subject to:

$$Q \geq 1 \tag{7}$$

$$Q < D \tag{8}$$

Eq. (7) shows that Q can never be $0 < Q < 1$ nor $Q \leq 0$. The former suggests no fractional Q and the latter also suggests nonnegative Q. The value of having (8) is to ignore the choice of having $Q^* = D$ since this case would translate (6) into a linear programming model and the problem is impractical to applications because for large values of D, and consequently Q, the manufacturing system can incur the risk of material shortage, machine breakdown, inadvertent conditions, etc. We need to determine Q^* to minimize $M(Q)$ given (8).

3 OPTIMAL STRATEGY

Consider the nonlinear optimization model in (6) with constraints (7) and (8). Suppose that the constraint task is $j = 2$ with task time t_2 with corresponding transactional time P_2 and material-handling time H_1. Equation (6) can be described as,

$$\min_Q M(Q) = \sum_{j=1}^{N}\{(P_j + Qt_j) + H_j\} + \left(\frac{D}{Q} - 1\right)\{(P_2 + Qt_2) + H_1\} \tag{9}$$

subject to:

$$-Q + 1 \leq 0 \tag{10}$$

$$Q - D < 0. \tag{11}$$

The Kuhn-Tucker necessary conditions for the stationary point of the program (9), (10) and (11) can be represented by the following,

$$(\lambda_1, \lambda_2) \leq 0 \tag{12}$$

$$\sum_{j=1}^{N} t_j + \left(\frac{D}{Q} - 1\right)t_2 - \frac{D}{Q^2}(P_2 + Qt_2 + H_1) - (\lambda_1, \lambda_2)\begin{bmatrix}-1\\1\end{bmatrix} = 0 \tag{13}$$

$$\lambda_1(-Q + 1) = 0 \tag{14}$$

$$\lambda_2(Q - D) = 0 \tag{15}$$

$$-Q + 1 \leq 0 \tag{16}$$

$$Q - D < 0. \tag{17}$$

These conditions can be simplified as,

$$(\lambda_1, \lambda_2) \leq 0 \tag{18}$$

$$\sum_{j=1}^{N} t_j + \left(\frac{D}{Q} - 1\right)t_2 - \frac{D}{Q^2}(P_2 + Qt_2 + H_1) + \lambda_1 - \lambda_2 = 0 \tag{19}$$

$$\lambda_1(-Q + 1) = 0 \tag{20}$$

$$\lambda_2(Q - D) = 0 \tag{21}$$

$$-Q + 1 \leq 0 \tag{22}$$

$$Q - D < 0 \tag{23}$$

where λ_1 and λ_2 are the Lagrange multiplier to the nonnegative constraint and the Lagrange multiplier to the nonlinearity constraint, respectively.

There are two feasible solutions for the equations presented in (18) through (23). This depends on the choice regarding (20). Suppose $-Q + 1 = 0$, which means that $Q^* = 1$ and $\lambda_1 \neq 0$. By (21), $Q - D \neq 0$ since it violates (23). Hence, we are prompted to accept $\lambda_2 = 0$. Therefore, the solution of the Kuhn-Tucker conditions would be $Q^* = 1$, $\lambda_1 \neq 0$ and $\lambda_2 = 0$. Now let have a look on the value of λ_1. By substitution to (19), we get,

$$\sum_{j=1}^{N} t_j + (D - 1)t_2 - D(P_2 + t_2 + H_1) + \lambda_1 = 0 \tag{24}$$

Since $\lambda_1 \leq 0$, then (24) can be written as follows,

$$\lambda_1 = D(P_2 + t_2 + H_1) - \sum_{j=1}^{N} t_j - (D - 1)t_2 \leq 0 \tag{25}$$

From (25), (18) can only be satisfied if,

$$D \leq \frac{\sum_{j=1, j\neq 2}^{N} t_j}{P_2 + H_1}. \tag{26}$$

Such case is trivial and can only be true for small values of D. In actual setting, (26) may not transpire. Hence the solution,

$$Q^* = 1 \tag{27}$$

$$\lambda_1 = D(P_2 + t_2 + H_1) - \sum_{j=1}^{N} t_j - (D - 1)t_2 \tag{28}$$

$$\lambda_2 = 0 \tag{29}$$

does not yield a global constrained minimum and may be discarded for this point. Moreover, letting $Q^* = 1$, the amount of P_2 and H_1 in (9) increases linearly by a factor of $(D - 1)$ which is unsatisfactory for $M(Q)$.

Another solution for (18) through (23) is to let $\lambda_1 = 0$ in (20) and $\lambda_2 = 0$ in (21) with similar explanation. This simplifies our problem into the following condition,

$$\sum_{j=1}^{N} t_j + \left(\frac{D}{Q} - 1\right)t_2 - \frac{D}{Q^2}(P_2 + Qt_2 + H_1) = 0, \tag{30}$$

which can be solved algebraically. Simplifying terms, we have,

$$\sum_{j=1}^{N} t_j + \frac{D}{Q}t_2 - t_2 - \frac{D}{Q^2}P_2 - \frac{D}{Q}t_2 - \frac{D}{Q^2}H_1 = 0. \tag{31}$$

This cancels out the term $\frac{D}{Q}t_2$ reduces the equation to,

$$-\frac{D}{Q^2}(P_2 + H_1) + \sum_{j=1, j\neq 2}^{N} t_j = 0 \tag{32}$$

$$\frac{-D(P_2 + H_1) + Q^2 \sum_{j=1, j\neq 2}^{N} t_j}{Q^2} = 0 \tag{33}$$

$$Q^2 \sum_{j=1, j\neq 2}^{N} t_j - D(P_2 + H_1) = 0. \tag{34}$$

Simplifying further, we have,

$$Q = \pm\sqrt{\frac{D(P_2 + H_1)}{\sum_{j=1, j\neq 2}^{N} t_j}}. \tag{35}$$

We take the positive value of Q which brings the solution to,

$$Q^* = \sqrt{\frac{D(P_2 + H_1)}{\sum_{j=1, j\neq 2}^{N} t_j}}. \tag{36}$$

The solution of (18) through (23) is $Q^* = \sqrt{\frac{D(P_2+H_1)}{\sum_{j=1, j\neq 2}^{N} t_j}}$, $\lambda_1 = \lambda_2 = 0$. It is not surprising for practical logic that $\lambda_1 = \lambda_2 = 0$ since by definition the Lagrange multipliers are sensitivity coefficients of the Jacobean method and described as the rate of variation of $M(Q)$ with respect to the constraints. As in our case, the constraints are basically not representing resources but rather, as delimitation for practical applications.

Therefore, any change in the constraints does not affect the makespan $M(Q)$.

When checking for the sufficient conditions for the Kuhn-Tucker conditions, by inspection, it can be observed that $M(Q)$ is a convex function for $Q \geq 0$, and (10) and (11) are convex sets, hence the resulting solution (stationary point) yields a global constrained minimum.

Q^* has interesting features. As observed, the optimal lot size is independent of the constraint task (or the longest task time), rather it is inversely proportional to the sum of the unconstraint task times. As $\sum_{\substack{j=1 \\ j\neq 2}}^{N} t_j$ increases, the lot size Q must decrease. Surprisingly, this relationship affirms ~~to~~ the widely accepted proposition e.g. in lean systems, in which lot size should be relatively small so that the cycle time (or completion time) of a lot should be also small. This protects the system from unexpected conditions in the manufacturing floor. Another important result of the paper is the proportionality relationship between the transactional time and material handling time (at constraint task) to the optimal lot size. As these respective times increase, Q must also increase proportionately to the power of $\frac{1}{2}$. This relationship poses economies of scale. Since transactional time and material handling time at constraint task are fix, the solution utilizes economies of scale so that these times are distributed over a larger Q resulting in low time per unit of output value. And lastly, Q^* depends on demand D, that is, as D is increased, Q^* is also increased. For practical applications, in a manufacturing setup, Q must be constant regardless of demand D for purpose of consistency to daily manufacturing operations. One approach of this concern is to replace the value of D in (36) by a planned weekly loading (in case, weekly). In this case $\frac{dQ^*}{dD} = 0$, this makes Q^* constant and consistent for everyday operations as long as $(P_2 + H_1)$ and $\sum_{\substack{j=1 \\ j\neq 2}}^{N} t_j$ do not change.

4 SENSITIVITY ANALYSIS

The solution $Q^* = \sqrt{\frac{D(P_2+H_1)}{\sum_{\substack{j=1 \\ j\neq 2}}^{N} t_j}}$, $\lambda_1 = \lambda_2 = 0$ practically suggests that any changes in the constraints do not affect the optimum. Another important contribution of this paper is the relationship exists in (26) and (36). Consequently, (26) and (36) are equal if we let $Q^* = 1$ in (36). This means that if $D \leq \frac{\sum_{\substack{j=1 \\ j\neq 2}}^{N} t_j}{P_2+H_1}$ in (36), the resulting $Q^* = 1$ and from $\lambda_1 = 0$ becomes $\lambda_1 = D(P_2 + t_2 + H_1) - \sum_{j=1}^{N} t_j - (D-1)t_2$. This faces us an interesting fact that if we still hold $Q^* = 1$ given that $D > \frac{\sum_{\substack{j=1 \\ j\neq 2}}^{N} t_j}{P_2+H_1}$, then $\lambda_1 > 0$, and every unit difference of $[Q^* - (Q = 1)]$ shall increase the manufacturing makespan by a factor of $\{D(P_2 + t_2 + H_1) - \sum_{j=1}^{N} t_j - (D-1)t_2\}$. This is one of the drawbacks of lean manufacturing system if lot size is forced to go as low as 1unit, considering the condition in (26).

5 CONCLUDING REMARKS

Considering a manufacturing system composed of task sequence $j = 1,2,3,\ldots,N$ having task times t_j, preparatory transactional times P_j, and material-handling time H_j moved from j to $j+1$, the minimum manufacturing makespan function $M(Q)$ in completing demand of quantity D as a lot size problem can be established. The resulting problem is a nonlinear constrained optimization model and can be solved using Kuhn-Tucker necessary and sufficiency conditions. The optimal lot size is a function of the demand, the transactional activities, the length of travel time at the constraint task, and the sum of unconstrained task time. One of the contributions of this paper is a method of minimizing makespan without shortening the constraint task. Another contribution is the proposition that the optimal lot size is not affected by the constraint task times, but rather the sum of the unconstraint task times. Lastly, this paper developed the necessary condition in having a lot size equal to one.

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