A Note on Minimizing the Total Loss for Two Quality Characteristics with One-sided Specification Limit

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Abstract: A single quality characteristic of a product is typically considered on the development of economic specification limits in the recent researches. However, from the view of the customers, products are often evaluated based on more than one quality characteristics. Due to the observation, this paper will not only develop an optimum process for two quality characteristics based on bivariate exponential distributions, but also provide a sensitivity analysis for the results.

Keywords: economic specification limits; two quality characteristics; optimum process

1 INTRODUCTION

In many production managements, the producers should concentrate on the obtained satisfaction measure to fulfill the customers' requirements and at the same time gain more profit (Fazlollahtabar et al. [1], Kumar [2]). Wen and Mergen [3] investigate find the best location for process mean for a situation where the process is stable but not capable of meeting the specification limits in the short term. They select the process mean based on balancing the cost of not meeting the upper specification limit and the lower specification limit. Furthermore, only single quality characteristic was discussed in their model. However, in practice, most process monitoring and control scenarios involve several related variables. Although applying univariate control charts to each individual variable is a possible solution in the literature, however, it will be seen that this is inefficient and can lead to erroneous conclusions.

Therefore, multivariate methods that consider the variables jointly are required (Montgomery [4]).

In the previous researches, most of researchers paid more attention to study process monitoring and control primarily from univariate perspective; that is, they assumed that there is only one process output variable or quality characteristic of interest. For example, Chen and Chou [5] proposed a modified Wen and Mergen's model including the quadratic quality loss for a one-sided specification limit. For more recently works, Matsuura [6] studied optimal partitioning of the dimensional distributions of the components in selective assembly which extends previous results for squared error loss function to cover general convex loss functions, including asymmetric convex loss

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functions. Considering the process capability indices (PCIs) used in statistical process control to evaluate the capability of the processes in satisfying the customer's needs. Jalili et al. [7] proposed a new multivariate PCI to analyze the processes with one or more unilateral specification limits. However, Kapur and Cho [8] further pointed out that there are many situations from the viewpoint of the customer in which more than one quality characteristics are necessary. They provided an example that the overall quality of a metal cutting tool includes several qualities such as cutting force, cutting speed, and metal removal rate. Hence, the simultaneous control of more than one correlated quality characteristic is often necessary. Moreover, the relative articles of more than one quality characteristics in the literature are very limited. Kapur and Cho [8] studied a multivariate quality loss function for setting economic specification limits. They obtained the optimal specification limits based on minimizing the total losses to society, including the variability from the target value, the scarp cost, and the inspection cost, etc. Chen and Chou [9] proposed a modified Wen and Mergen's cost model with bivariate quality loss, for a product within specifications, for determining the optimum process mean. Motivated by this shortcoming in the literature, this study will attempt to propose a modified Wen and Mergen's cost model with bivariate quality characteristics for determining the optimum process mean.

2 WEN AND MERGEN'S COST MODEL FOR A SINGLE QUALITY CHARACTERISTIC

There are three assumptions in Wen and Mergen's model. The details are provided as follows;

1. The quality characteristic, *X*, is normally distributed with an unknown mean μ and a known variance σ^2.
2. The quality characteristic is the nominal-is-best.
3. The target value, *T*, is the middle value of the specification, i.e., $T = (T_U + T_L)/2$.

According to the above assumptions, the total loss per item is developed in the following;

$$C_T = D_U \int_{T_U}^{\infty} f(x)dx + D_L \int_{-\infty}^{T_L} f(x)dx \quad (1)$$

where,

T_U = the upper specification limit,

T_L = the lower specification limit,

C_T = the total loss per item due to exceeding the T_U and T_L,

D_U = the monetary loss per item of exceeding T_U,

D_L = the monetary loss per item of exceeding T_L,

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \ -\infty < x < \infty .$$

3 MODIFIED WEN AND MERGEN'S COST MODEL FOR TWO SINGLE QUALITY CHARACTERISTICS

Similar to the concept of the quality evaluation system, a product is classified as nonconforming if the quality characteristic of the product fails to meet the predetermined specification limits and then a certain amount of economic loss is incurred, otherwise, it is classified as conforming and no loss is incurred. The loss function approach can be used effectively to determine whether 100-percent inspection can be justified or not. It should be noted that the objective of inspection is to screen or repair defective products that cannot meet the given specifications. There are common used three types of quality characteristic for the loss function; nominal the best, smaller the letter, and larger the better. For the larger the better characteristic, there are cases where the-larger-the better is applicable to characteristics such as the strength of materials and fuel efficiency. In these cases, there no predetermined target values, and the larger the value of the characteristic, the better it is. The situation also implies that zero loss is attained when the target is infinity. Therefore, the lower specification limit is generally assumed in case the producers design specifications for this type of quality characteristic. Therefore, in this section, we consider that quality characteristics *X* and *Y* obey the bivariate exponential distributions with a joint probability density function with a known associate relationship θ given by

$$f(x,y)=\lambda_1\lambda_2[(1+\lambda_1\theta x)(1+\lambda_2\theta y)-\theta]\cdot e^{-(\lambda_1 x+\lambda_2 y+\lambda_1\lambda_2\theta x y)},\ 0\le\theta\le 1,\ x>0,\ y>0,$$

where λ_1 and λ_2 denote the process rate for quality characteristic *x* and *y*, respectively (Anian and Beg [10]).

According to equation (1), the total loss per item is

$$\begin{aligned}C_T(\lambda_1,\lambda_2)&=D_x\int_0^{T_x}\int_0^{\infty}f(x,y)\,dy\,dx\\&+D_y\int_0^{T_y}\int_0^{\infty}f(x,y)\,dx\,dy\\&-D_{xy}\int_0^{T_x}\int_0^{T_y}f(x,y)\,dy\,dx\\&=D_x(1-e^{-\lambda_1T_x})+D_y(1-e^{-\lambda_2T_y})-D_{xy}\\&\quad(1-e^{-\lambda_1T_x}-e^{-\lambda_2T_y}+e^{-(\lambda_1T_x+\lambda_2T_y+\lambda_1\lambda_2\theta T_xT_y)})\end{aligned}\tag{2}$$

where,

T_x = the lower specification limit for quality characteristic *X*,

T_y = the upper specification limit for quality characteristic *Y*,

D_x = the monetary loss per item of exceeding T_x for quality characteristic *X*,

D_y = the monetary loss per item of exceeding T_y for quality characteristic *Y*,

D_{xy} = the monetary loss per item of exceeding T_x and T_y for quality characteristics *X* and *Y*.

In order to minimize $C_T(\lambda_1,\lambda_2)$, we differentiate equation (2) with respective to parameters λ_1 and λ_2, and get the following expressions:

$$T_xD_xe^{-\lambda_1T_x}-T_xD_{xy}e^{-\lambda_1T_x}+D_{xy}(T_x+T_xT_y\theta\lambda_2)\cdot e^{-(\lambda_1T_x+\lambda_2T_y+\lambda_1\lambda_2\theta T_xT_y)}=0,$$

and

$$T_yD_ye^{-\lambda_2T_y}-T_yD_{xy}e^{-\lambda_2T_y}+D_{xy}(T_y+T_xT_y\theta\lambda_1)\cdot e^{-(\lambda_1T_x+\lambda_2T_y+\lambda_1\lambda_2\theta T_xT_y)}=0.$$

After simplification, they reduce to

$$(1+T_y\theta\lambda_2)e^{-\lambda_2T_y(1+\lambda_1\theta T_x)}=\frac{D_{xy}-D_x}{D_{xy}},\tag{3}$$

and

$$(1+T_x\theta\lambda_1)e^{-\lambda_1T_x(1+\lambda_2\theta T_y)}=\frac{D_{xy}-D_y}{D_{xy}}.\tag{4}$$

The exact solutions for equations (3) and (4) are not easily to be solved. However, the approximate solution can be obtained by using software package.

For further reducing the problem, we assume that associate relationship θ is equal to zero, that is, quality characteristics *X* and *Y* obey two independent exponential distributions. The problem can be rewritten as follow;

$$\begin{aligned}C_T(\lambda_1,\lambda_2)&=D_x\int_0^{T_x}\int_0^{\infty}f(x,y)dydx\\&+D_y\int_0^{T_y}\int_0^{\infty}f(x,y)dxdy\\&-D_{xy}\int_0^{T_x}\int_0^{T_y}f(x,y)\,dy\,dx\\&=D_x\int_0^{T_x}\int_0^{\infty}\lambda_1\lambda_2e^{-(\lambda_1x+\lambda_2y)}dydx+\\&\quad D_y\int_0^{T_y}\int_0^{\infty}\lambda_1\lambda_2e^{-(\lambda_1x+\lambda_2y)}dxdy-\\&\quad D_{xy}\int_0^{T_x}\int_0^{T_y}\lambda_1\lambda_2e^{-(\lambda_1x+\lambda_2y)}dy\,dx\\&=D_x(1-e^{-\lambda_1T_x})+D_y(1-e^{-\lambda_2T_y})-\\&\quad D_{xy}(1-e^{-\lambda_1T_x})(1-e^{-\lambda_2T_y})\end{aligned}\tag{5}$$

Similarly, we differentiate equation (5) with respective to parameters λ_1 and λ_2, and get the following expressions:

$$D_xT_xe^{-\lambda_1T_x}-D_{xy}T_xe^{-\lambda_1T_x}(1-e^{-\lambda_2T_y})=0,$$

and

$$D_y T_y e^{-\lambda_2 T_y} - D_{xy} T_y e^{-\lambda_2 T_y}(1 - e^{-\lambda_1 T_x}) = 0 .$$

After simplification, the above equations reduce to

$$e^{-\lambda_2 T_y} = \frac{D_{xy} - D_x}{D_{xy}}, \tag{6}$$

and

$$e^{-\lambda_1 T_x} = \frac{D_{xy} - D_y}{D_{xy}}. \tag{7}$$

After solving for parameters λ_1 and λ_2, the solutions $\hat{\lambda}_1 = \frac{1}{T_x} \ln \frac{D_{xy}}{D_{xy} - D_y}$ and $\hat{\lambda}_2 = \frac{1}{T_y} \ln \frac{D_{xy}}{D_{xy} - D_x}$.

Thus, $\frac{D_x D_y}{D_{xy}}$ is the optimal value.

4 CONCLUSIONS

This paper modifies Wen and Mergen's cost model for a two-quality characteristic problem. The multivariate method is to determine the process means at an optimum level to minimize the total cost. The method follows Wen and Mergen's model which is useful for minimizing the quality loss in the short term until the process capability is improved. The main result is that for the general bivariate exponential distributions with a joint probability density function with a known associate relationship θ , the numerical solutions can be obtained from software package. Furthermore, the close form solution is proposed for two independent exponential distributions. Apart, the sensitivity analysis is also discussed.

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